King's College London

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

Degree Programmes	BSc, MSci, BEng, MEng
Module Code	5CCS2FC2
Module Title	Foundations of Computing II
Examination Period	January 2019 (Period 1)
Time Allowed	Two hours
Rubric	ANSWER ALL OF QUESTIONS 1–10, AND ANSWER TWO OF THREE FROM QUESTIONS 11–13. Questions 1–10 carry a total of FIFTY marks and each have ONE or MORE correct choices. In order to obtain full marks you must select all correct choices and only those. **Marks will be deducted for incorrect choices selected in those questions.** Questions 11–13 each carry TWENTY FIVE marks. The answers to questions **1–10** need to be clearly made by pen on the appropriate grid on the **answer sheet** provided at the back of the exam paper. The answers to questions **11–13** need to be written by pen on the separate **answer book** provided.
Calculators	Calculators may be used. The following models are permitted: Casio fx83 / Casio fx85.
Notes	Books, notes or other written material may not be brought into this examination

PLEASE DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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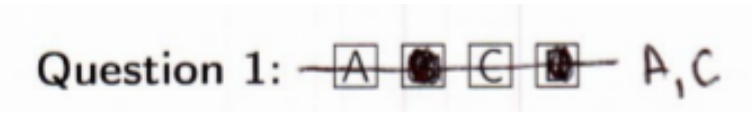
How to correctly use your Answer Sheet and Answer Book

- Your **answer book** must be used to provide the answers to Questions 11, 12 and 13.
- Your **answer book** may also be used for rough work for Questions 1–10. However you must clearly indicate on the answer book where the answer to Questions 11, 12 and 13 are located.
- You must provide the answers to Questions 1–10 on the **answer sheet** by making the appropriate choice selections when ready to commit them.
- Only the answers to Questions 1–10 provided in the answer sheet will be considered.
- Please make your choice selections on the **answer sheet** by filling out the corresponding box using a **pen**.
- Below are examples of how to correctly and incorrectly make your choice selections on the answer sheet.

Correct	Incorrect
Question 1: ■ B C D	Question 1: ~~A~~ B C D

- Only one answer sheet per exam will be provided so make your choice selections carefully.
- If for any reason you make a mistake making your choice selection, put a line over the choice boxes of the question item and provide the correction in writing next to the question item. In the example below, the selections A and C will be considered instead of B and D.



1. Which of the following four statements accurately describe what it means for a decision problem X to be recursively enumerable? Select ONE correct answer.

A. There exists a sound and complete algoritm for X.

B. Every algorithm for X is both sound and complete.

C. Any sound and complete algorithm for X must also be terminating.

D. There are no sound, complete and terminating algorithms for X.

[4 marks]

2. Which of the following accurately describes the Cook-Levin Theorem? Select ONE correct answer.

A. Every Turing Machine problem can be polynomially reduced to an instance of SAT.

B. Every problem that can be verified in polynomial time can be solved in polynomial time.

C. The Halting problem for Turing Machines is undecidable.

D. The Boolean Satisfiability problem SAT is NP-complete.

[4 marks]

3. Using the Master Theorem, determine which of the following recursion relations decribe functions whose growth rate is $\Theta(n \log n)$. In all cases, you may assume that $T(1) = 1$. Select ALL correct answers.

A. $T(n) = 4\,T(n/2) + n \log n$

B. $T(n) = T(n/4) + n \log n$

C. $T(n) = 5\,T(n/5) + n$

D. $T(n) = 5\,T(n/2) + n^3$

[6 marks]

4. Consider the following instance of 2SAT:

$$(\neg Q \vee P) \wedge (\neg P \vee \neg S) \wedge (\neg P \vee \neg R) \wedge (S \vee R) \wedge (Q \vee R)$$

Draw the implication graph and identify the strongly connected components. Which of the following subsets are strongly connected components of the implication graph? Select ALL correct answers.

A. $\{P, Q, \neg R, S\}$

B. $\{\neg S\}$

C. $\{\neg P, \neg R, \neg S\}$

D. $\{\neg P, Q, S\}$

E. $\{\neg P, \neg Q, R\}$

[4 marks]

5. Use the implication graph constructed in your previous answer to determine whether the formula in Question 4 is satisfiable.

A. Satisfiable

B. Unsatisfiable

[2 marks]

6. Consider the following tableau:

$T_1 =$

x	y	s_1	s_2	s_3	C	
-9	-5	1	0	0	0	9
2	-6	0	1	0	0	8
-3	10	0	0	1	0	7
-9	-7	0	0	0	1	0

Identify the pivot and perform ONE iteration of the simplex method on the initial tableau T_1 to generate a second tableau T_2.

From your second tableau T_2, what are the values of X, Y, Z and W, as located in the following positions:

$T_2 =$

x	y	s_1	s_2	s_3	C	
$-$	$-$	$-$	Z	$-$	$-$	$-$
$-$	$-$	$-$	$-$	$-$	$-$	W
X	$-$	$-$	$-$	$-$	$-$	$-$
$-$	Y	$-$	$-$	$-$	$-$	$-$

Select ONE correct answer.

A. $X = -0.3$, $Y = 0$, $Z = 0$, $W = 9.8$

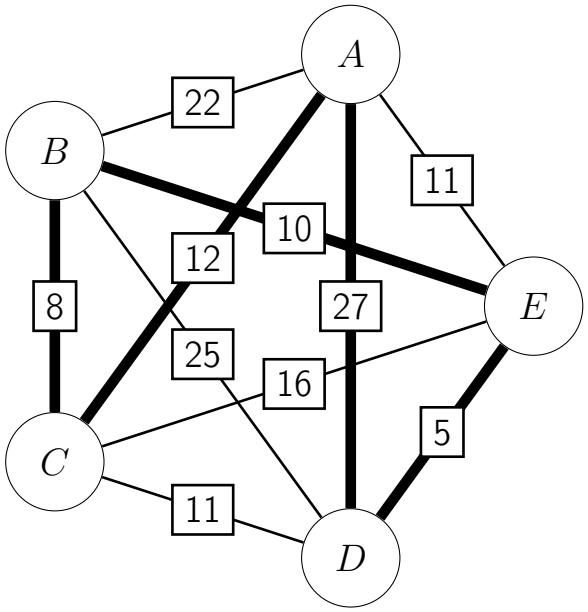
B. $X = 0$, $Y = -34$, $Z = 4.5$, $W = 4$

C. $X = 0$, $Y = -2$, $Z = 0$, $W = 6$

D. $X = 0$, $Y = 0$, $Z = 0$, $W = 1$

[7 marks]

7. Consider the following instance of the Travelling Salesman Problem:



Highlighted in bold is the following route of length 62:

$$A \to C \to B \to E \to D \to A$$

Using the 2-opt algorithm, which of the following modifications are valid swaps that can be made to the above route to obtain a shorter route? Select ALL correct answers.

A. Remove $(A, D), (C, B)$ and Add $(C, D), (A, B)$.

B. Remove $(A, C), (B, E)$ and Add $(A, B), (C, E)$.

C. Remove $(A, C), (A, D)$ and add $(C, D), (A, A)$.

D. Remove $(A, D), (B, E)$ and add $(A, B), (D, E)$.

E. Remove $(A, D), (B, E)$ and add $(A, E), (B, D)$.

[6 marks]

8. What does it mean to say that an optimisation problem X is r-approximable, where $r \geq 1$? Select ONE correct answer.

A. There is no exact algorithm for X that runs in polynomial-time.

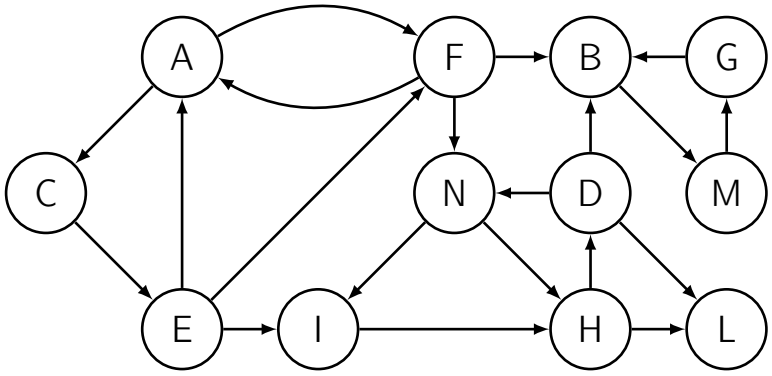
B. These exists a polynomial-time algorithm for X than returns an optimal solution after at most r iterations.

C. Every polynomial-time approximate algorithm for X has an approximation ratio greater than or equal to r.

D. There exists a polynomial-time algorithm for X than returns an approximate solution no worse than r times the global optimum.

[4 marks]

9. Consider the following dependency graph:



Construct the component graph by identifying each of the strongly connected components (SCCs).

Which of the following edges CAN be safely added to the dependency graph without reducing the number of strongly connected components? Select ALL correct answers.

A. Add (L, A).

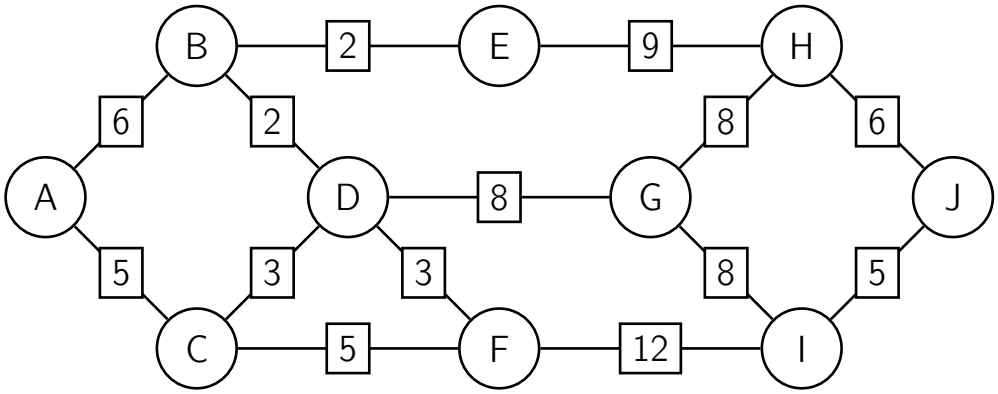
B. Add (M, L).

C. Add (C, F).

D. Add (M, D).

[8 marks]

10. Consider the following weighted graph G on ten vertices:



Use either Kruskal's or Prim's algorithm to construct a minimum spanning tree for G. What is the weight of the minimum spanning tree? Select ONE correct answer.

A. 37

B. 39

C. 42

D. 44

[5 marks]

Please answer TWO out of THREE from Questions 11–13 in the separate answer book provided.

11. a. Show that the following language is undecidable

$$\mathsf{REJECT}_{TM} \;=\; \{\langle M, w\rangle : M \text{ encodes a TM that rejects } w\}.$$

from first principles (*i.e.* without using a reduction from another known undecidable problem).

[13 marks]

b. Show that REJECT_{TM} is recursively enumerable by constructing a sound and complete algorithm that recognises all words $\langle M, w\rangle \in \mathsf{REJECT}_{TM}$, where M encodes a TM that rejects w.

[7 marks]

c. Hence, or otherwise, show that the complement $\overline{\mathsf{REJECT}_{TM}}$ is *not* recursively enumerable.

[5 marks]

12. a. Consider the following recursion relation

$$\begin{aligned} T(0) &= 0 \\ T(1) &= 1 \\ T(n+1) &= T(n) + 2\,T(n-1) \end{aligned}$$

for all $n \geq 1$. Prove, by induction on the input n, that

$$T(n) = \frac{2^n - (-1)^n}{3}$$

for all $n \geq 0$. You should clearly state your induction hypothesis, base case(s) and induction step.

[10 marks]

b. Consider the following set of propositional clauses (numbered for reference):

$$(\neg P \vee R) \quad (1)$$
$$(\neg P \vee Q \vee T) \quad (2)$$
$$(Q \vee \neg R \vee \neg T) \quad (3)$$
$$(P \vee R \vee S) \quad (4)$$
$$(P \vee \neg R) \quad (5)$$
$$(P \vee R \vee \neg S) \quad (6)$$
$$(\neg P \vee \neg Q) \quad (7)$$

Use the *Davis-Putnam-Logemann-Loveland (DPLL)* algorithm to determine whether the above set of clauses is satisfiable. You should state which rule is being applied in each instance.

[10 marks]

QUESTION 12 CONTINUES ON NEXT PAGE

c. Prove that SAT is polynomially reducible to 3SAT by describing a polynomial-time algorithm for converting an instance of SAT into an instance of 3SAT. You should explain why your algorithm terminates.

[5 marks]

13. The problem CLIQUE takes as input a graph $G = (V, E)$ and integer $k > 0$ and returns `True` if there is a set $C \subseteq V$ of size k such that every pair of vertices in C are connected with an edge from E.

a. What does it mean to say that the Boolean Satisfiability problem SAT is *polynomially reducible* to CLIQUE?

[4 marks]

b. Given the following input formula

$$F \;=\; (\neg P \vee Q \vee \neg R) \wedge (P \vee Q) \wedge (\neg Q \vee R)$$

construct a graph G_F such that F is satisfiable if and only if G_F has a clique of size $k = 3$.

[9 marks]

c. Explain why the existence of a clique of size $k = 3$ guarantees that there is a satisfying assignment for F?

[6 marks]

d. Describe a non-deterministic algorithm for CLIQUE that runs in polynomial time, *i.e.*, show that CLIQUE belongs to the class NP.

[6 marks]

+1/9/50+

5CCS2FC2 Foundations of Computing II
Answer sheet (January 2019)

Please encode the 5 digits of your candidate number ignoring its first letter on the **grid** on the left and for extra security write your candidate number and your desk number in the box on the right. Use a black or blue pen, but not a pencil.

0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Candidate number:

. . . . . .

Desk number:

. . . . . .

Please answer ALL of Questions 1–10 in the grid below:

Question 1: A B C D

Question 2: A B C D

Question 3: A B C D

Question 4: A B C D E

Question 5: A B

Question 6: A B C D

Question 7: A B C D E

Question 8: A B C D

Question 9: A B C D

Question 10: A B C D

Please answer TWO out of THREE from Questions 11–13 in the separate answer book provided.