King's College London

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

Degree Programmes	BSc, MSci, BEng, MEng
Module Code	5CCS2FC2
Module Title	Foundations of Computing II
Examination Period	January 2020 (Period 1)
Time Allowed	Two hours
Rubric	ANSWER ALL QUESTIONS 1–10, AND ANSWER TWO OF QUESTIONS 11–13. Questions 1–10 carry a total of FIFTY marks and each have ONE or MORE correct choices. In order to obtain full marks you must select all correct choices and only those. **Marks will be deducted for incorrect choices selected in those questions.** Questions 11–13 each carry TWENTY FIVE marks. The answers to questions **1–10** need to be clearly made by pen on the appropriate grid on the **answer sheet** provided at the back of the exam paper. The answers to questions **11–13** need to be written by pen on the separate **answer book** provided.
Calculators	Calculators may be used. The following models are permitted: Casio fx83 / Casio fx85.
Notes	Books, notes or other written material may not be brought into this examination

PLEASE DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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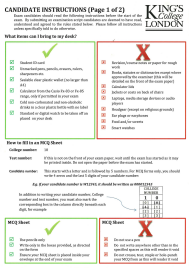
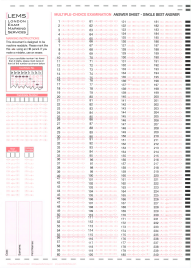
How to correctly use your Answer Sheet and Answer Book

- Your **answer book** must be used to provide the answers to Questions 11, 12 and 13. Your **answer book** may also be used for rough work for Questions 1–10. However you must clearly indicate on the answer book where the answer to Questions 11, 12 and 13 are.
- You must provide the answers to Questions 1–10 on the **answer sheet** by making the appropriate choice selections when ready to commit them. Only the answers to Questions 1–10 provided in the answer sheet will be considered.
- Please make your choice selections on the **answer sheet** (**at the back of the exam paper**) by filling out the corresponding box using a **pen**.
- If you find the following 'Candidate Instructions' and 'LEMS answer sheet' on your desk, **please ignore them:**





- Below are examples of how to correctly and incorrectly make your choice selections on the answer sheet.

Correct	Incorrect
Question 1: ■ B C D	Question 1: ~~A~~ B C D

- Only one answer sheet per exam will be provided so make your choice selections carefully.
- If for any reason you make a mistake making your choice selection, put a line over the choice boxes of the question item and provide the correction in writing next to the question item. In the example below, the selections A and C will be considered instead of B and D.

Question 1: ~~A~~ ~~■~~ ~~C~~ ~~■~~ A, C

1. Which of the following four statements accurately describe what it means for a decision problem X to be recursively enumerable? Select ONE correct answer.

A. There is no algorithm for X that is sound, complete, and terminating.

B. The complement $\overline{X}$ is decidable.

C. The complement $\overline{X}$ is *not* recursively enumerable.

D. There is some algorithm for X that is both sound and complete.

E. None of the above.

[4 marks]

2. Let X and Y be decision problems over an alphabet Σ. What are the conditions for a function $f : \Sigma^* \to \Sigma^*$ to be a *mapping reduction* from X to Y? Select ALL conditions that apply.

A. $f(w) \in X$ if and only if $f(w) \in Y$.

B. $w \in X$ if and only if $f(w) \in Y$.

C. f is a polynomial function.

D. f is a computable function.

E. X is undecidable.

[6 marks]

3. Using the Master Theorem, determine which of the following recursion relations decribe functions whose growth rate is $\Theta(n^2 \log n)$. In all cases, you may assume that $T(1) = 1$. Select ALL correct answers.

A. $T(n) = 4\,T(n/2) + n \log_2 n$

B. $T(n) = 4\,T(n/2) + n^2 \log_2 n$

C. $T(n) = 9\,T(n/3) + n(n+1)$

D. $T(n) = 2\,T(n/4) + 3n^2$

E. None of the above.

[6 marks]

4. Consider the following instance of 2SAT:

$$(P \vee Q) \wedge (P \vee \neg S) \wedge (\neg P \vee \neg R) \wedge (R \vee Q) \wedge (R \vee S)$$

How many strongly connected components are there in the implication graph for this formula? Select ONE correct answer.

A. 1

B. 2

C. 3

D. 4

E. 5 or more

[3 marks]

5. Which of the following conditions on the implication graph is necessary and sufficient for an instance of 2SAT to be satisfiable? Select ONE correct answer.

A. There are no paths between any literal and its negation.

B. Every strongly connected component either only contains positive literals or only contains negative literals.

C. There are no cycles that contain both a literal and its negation.

D. Every strongly connected component is non-empty.

E. None of the above.

[3 marks]

6. What does it mean to say that an optimisation problem X is *unapproximable* (assuming P $\neq$ NP)? Select ONE correct answer.

A. There is no polynomial-time algorithm for X than returns an optimal solution after at most r iterations,

B. There exists a polynomial-time approximation algorithm for X with an approximation ratio $r < 1$.

C. There is no polynomial-time approximation algorithm for X with an approximation ratio $r \geq 1$, for any fixed r.

D. The decision problem associated with X is undecidable.

E. None of the above.

[4 marks]

7. Consider the following tableau:

$T_1 =$

x	y	s_1	s_2	s_3	C	
7	3	1	0	0	0	-8
1	-8	0	1	0	0	5
1	-7	0	0	1	0	9
-3	1	0	0	0	1	0

Identify the pivot and perform ONE iteration of the simplex method on the initial tableau T_1 to generate a second tableau T_2.

From your second tableau T_2, what are the values of X, Y, Z and W, as located in the following positions:

$T_2 =$

x	y	s_1	s_2	s_3	C	
$-$	$-$	$-$	Z	$-$	$-$	$-$
X	$-$	$-$	$-$	$-$	$-$	$-$
$-$	$-$	$-$	$-$	$-$	$-$	W
$-$	Y	$-$	$-$	$-$	$-$	$-$

Select ONE correct answer.

A. $X = -6/7$, $Y = 0$, $Z = -3$, $W = -9/7$

B. $X = 0$, $Y = 2/7$, $Z = 0$, $W = 59/7$

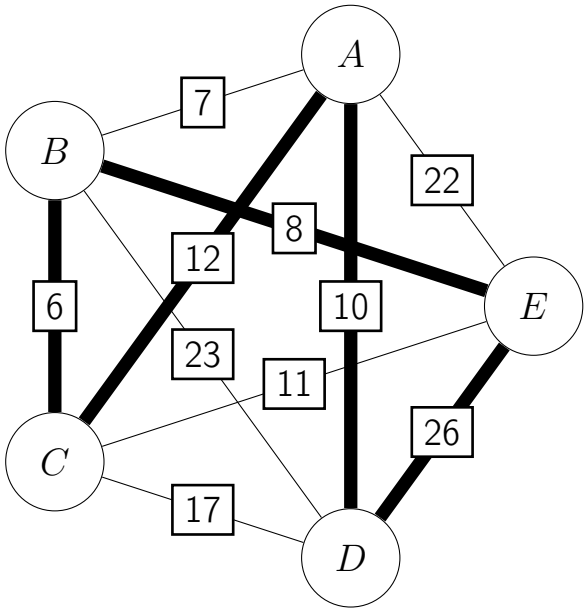
C. $X = 1$, $Y = -23$, $Z = -7$, $W = 4$

D. $X = 0$, $Y = -7$, $Z = 1$, $W = 59$

E. None of the above.

[7 marks]

8. Consider the following instance of the Travelling Salesman Problem:



Highlighted in bold is the following route of length 62:

$$A \to C \to B \to E \to D \to A$$

Apply the 2-opt algorithm until you reach a local minimum. How many applications of the 2-opt swap rule are required until you reach a local minimum? Select ONE correct answer

A. 1

B. 2

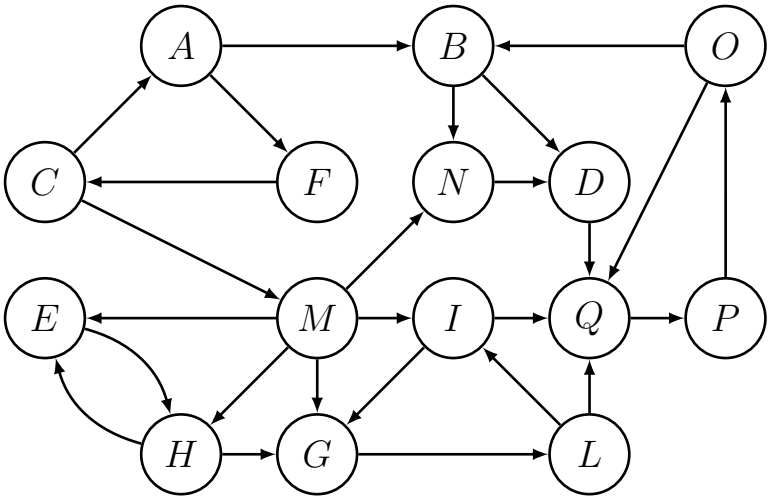
C. 3

D. 4

E. 5 or more

[6 marks]

9. Consider the following dependency graph:



Apply the strongly connected component (SCC) algorithm to identify all the SCCs.

Which is the last SCC to be identified by the algorithm? Select ONE correct answer.

A. $\{E, H\}$

B. $\{M\}$

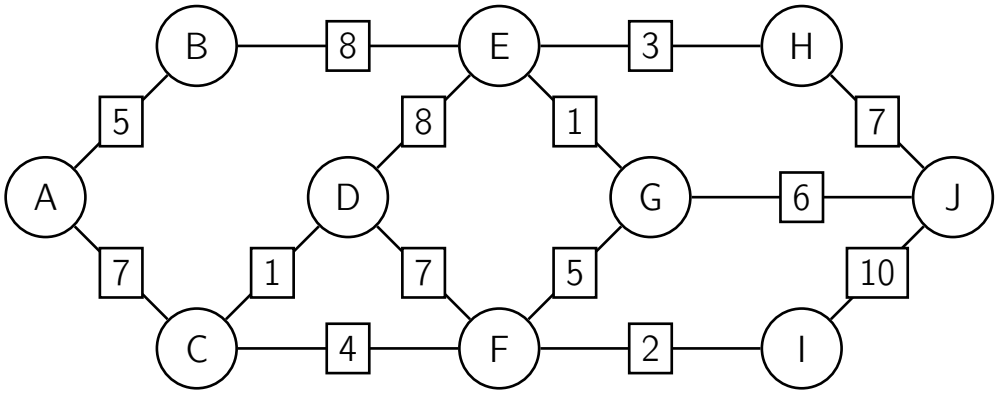
C. $\{A, C, F\}$

D. $\{B, D, N, O, P, Q\}$

E. None of the above.

[6 marks]

10. Consider the following weighted graph G on ten vertices:



Construct a minimum spanning tree for G by applying *Prim's algorithm* starting at vertex A. In which step is the edge (E, G) added to the spanning tree? Select ONE correct answer.

A. 1

B. 3

C. 5

D. 7

E. None of the above.

[5 marks]

Please answer TWO questions from Questions 11–13 on the separate answer book provided.

11. Consider the following language of all encodings of Turing Machines whose languages are non-empty; *i.e.* they accept at least one word

$$\overline{\mathsf{E}_{TM}} \;=\; \{\langle M\rangle : M \text{ encodes a TM such that } L(M) \neq \emptyset\}.$$

a. Show $\overline{\mathsf{E}_{TM}}$ is undecidable by constructing a mapping reduction from the language A_{TM}, where

$$\mathsf{A}_{TM} \;=\; \{\langle M, w\rangle : M \text{ encodes a TM that accepts the string } w\}.$$

You may assume that A_{TM} is known to be undecidable.

[13 marks]

b. Show that $\overline{\mathsf{E}_{TM}}$ is recursively enumerable by constructing a sound and complete algorithm that recognises all words $\langle M\rangle \in \overline{\mathsf{E}_{TM}}$.

[7 marks]

c. Hence, or otherwise, show that the complement E_{TM} is *not* recursively enumerable.

[5 marks]

12. a. Consider the following recurrence relation

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(\lceil n/3 \rceil) + 2 \end{aligned}$$

for all $n > 1$. Prove, by induction on the input n, that

$$T(n) \geq \log_2(n)$$

for all $n \geq 1$. You should clearly state your induction hypothesis, base case(s) and induction step.

[10 marks]

b. Consider the following set of propositional clauses (numbered for reference):

$$(Q \vee \neg S \vee \neg T) \quad (1)$$
$$(\neg Q \vee \neg T) \quad (2)$$
$$(\neg P \vee R \vee \neg S \vee \neg T) \quad (3)$$
$$(P \vee Q \vee S) \quad (4)$$
$$(P \vee \neg S \vee T) \quad (5)$$
$$(\neg P \vee Q) \quad (6)$$
$$(\neg Q \vee T) \quad (7)$$

Use the *Davis-Putnam-Logemann-Loveland (DPLL)* algorithm to determine whether the above set of clauses is satisfiable. You should state which rule is being applied in each instance.

[10 marks]

c. Describe a Divide-and-Conquer algorithm for a problem of your choice. Marks will be awarded for clarity.

[5 marks]

13. The problem HAMILTONIAN CYCLE takes as input a graph $G = (V, E)$ and returns `True` if there is cycle that visits each vertex exactly once.

a. Show that the HAMILTONIAN CYCLE problem belongs to the class NP by describing a non-deterministic algorithm for HAMILTONIAN CYCLE that runs in polynomial time.

[6 marks]

b. Given a propositional formula F, describe a procedure to construct a graph G_F such that F is satisfiable if and only if G_F contains a Hamiltonian cycle.

[9 marks]

c. Explain why the satsifiability of F guarantees that G_F contains a Hamiltonian cycle?

[5 marks]

d. Explain why the existence of a Hamiltonian cycle in G_F guarantees that F is satisfiable?

[5 marks]

+1/8/51+

5CCS2FC2 Foundations of Computing II
Answer sheet (January 2020)

Please encode the 5 digits of your candidate number ignoring its first letter on the **grid** on the left and for extra security write your candidate number and your desk number in the box on the right. Use a black or blue pen, but not a pencil.

0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Candidate number:

.....

Desk number:

.....

Please answer Questions 1–10 in the grid below:

Question 1:	A	B	C	D	E	**Question 6:**	A	B	C	D	E
Question 2:	A	B	C	D	E	**Question 7:**	A	B	C	D	E
Question 3:	A	B	C	D	E	**Question 8:**	A	B	C	D	E
Question 4:	A	B	C	D	E	**Question 9:**	A	B	C	D	E
Question 5:	A	B	C	D	E	**Question 10:**	A	B	C	D	E

Please answer TWO questions from Questions 11–13 in the separate answer book provided.