5CCS2FC2: Foundations of Computing II

Automata and Turing Machines

Week 1

Dr Christopher Hampson

Department of Informatics

King's College London

Not all languages are regular

Theorem The language $L = \{a^n b^n : n = 0, 1, 2, \ldots\}$ is *not* regular.

Proof:

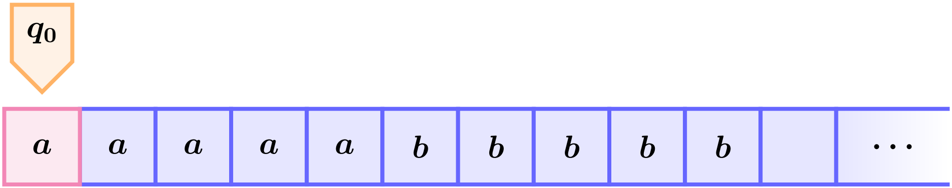
Step 1) Suppose, to the contrary that there is some DFA $A^\star$ such that

$$\textbf{Language}\,(A^\star) \;=\; \{a^n b^n \;:\; n = 0, 1, 2, 3, \ldots\}$$

Step 2) Let $p =$ number of states in $A^\star$

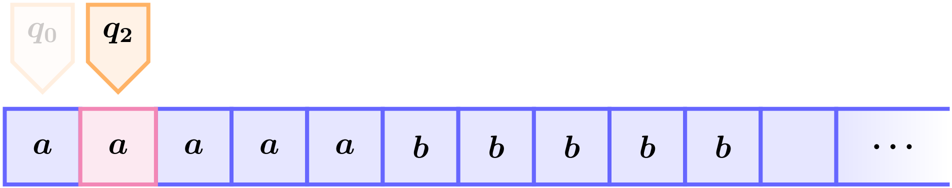
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



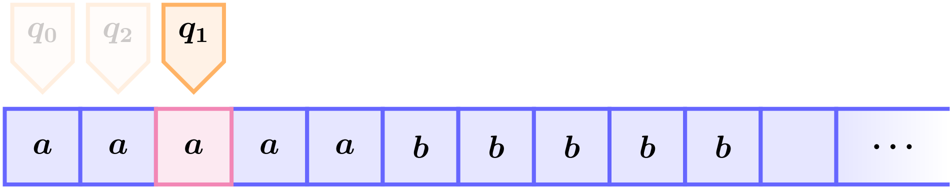
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



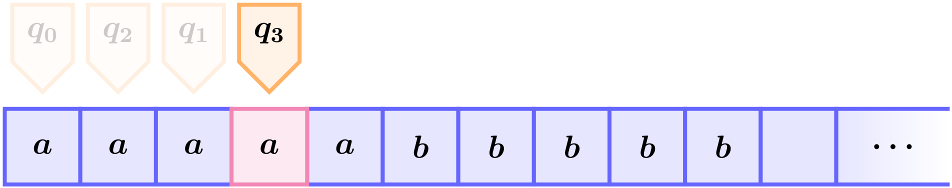
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



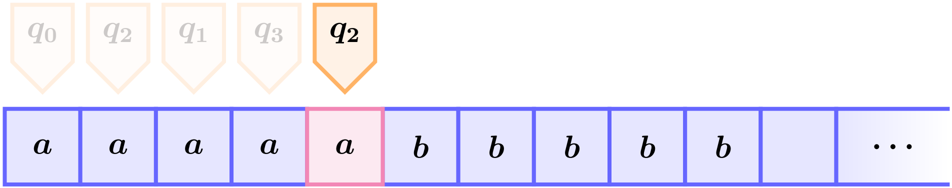
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



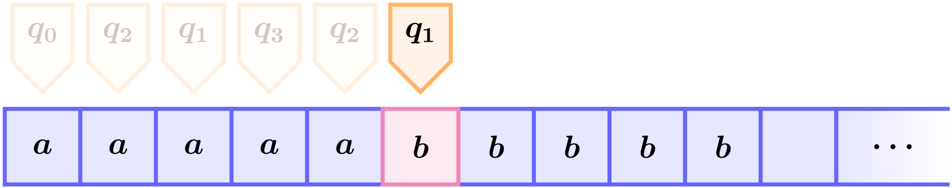
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



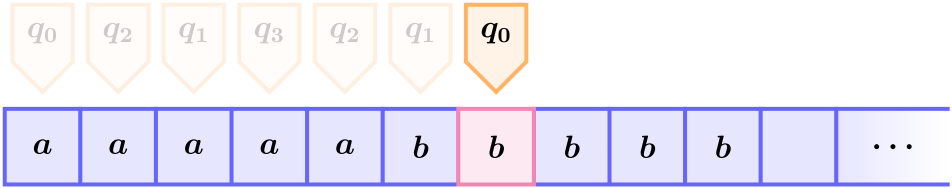
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



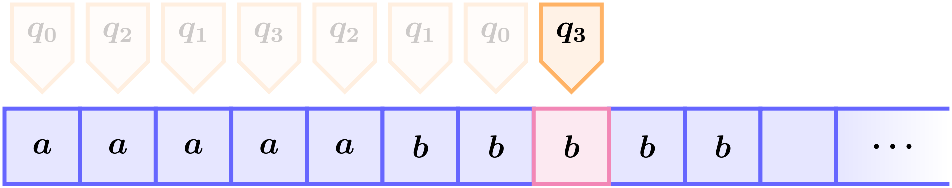
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



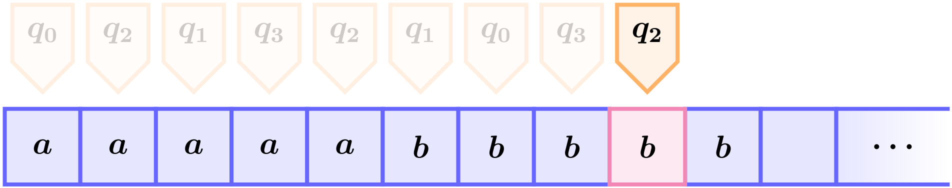
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



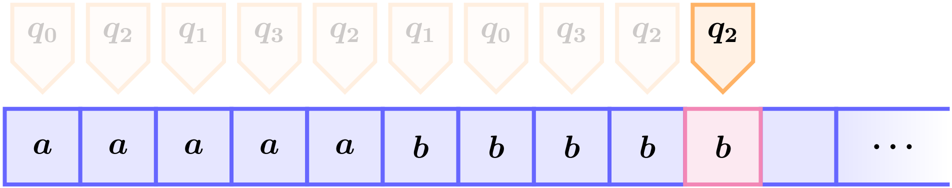
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



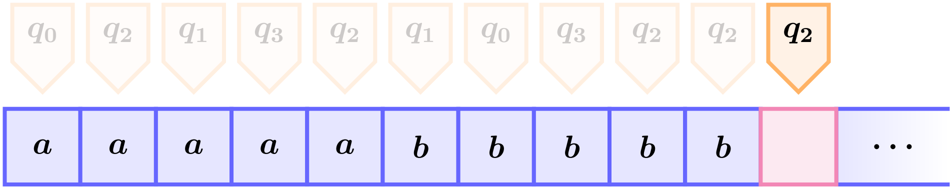
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



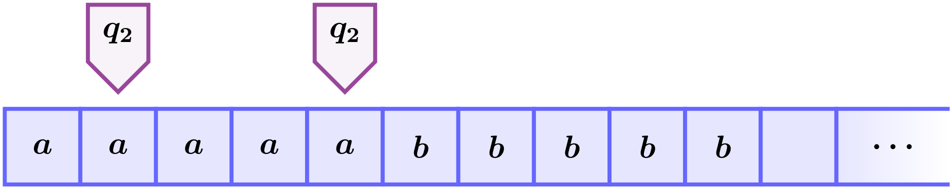
Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



Not all languages are regular

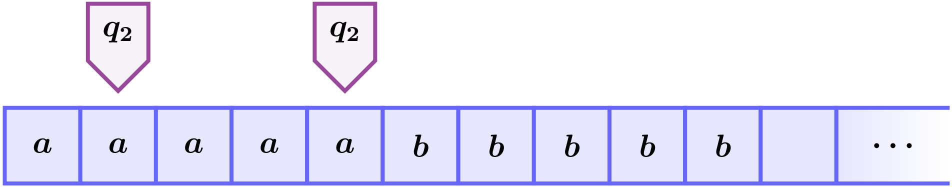
Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



$A^\star$ must **revisit** some state more than once while reading a's!

Not all languages are regular

Step 3) Consider what the computation of $a^k b^k$ looks like for $k > p$:



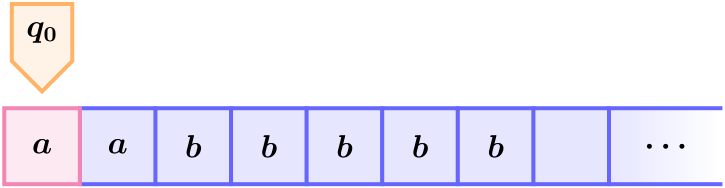
$A^\star$ must **revisit** some state more than once while reading a's!

Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:

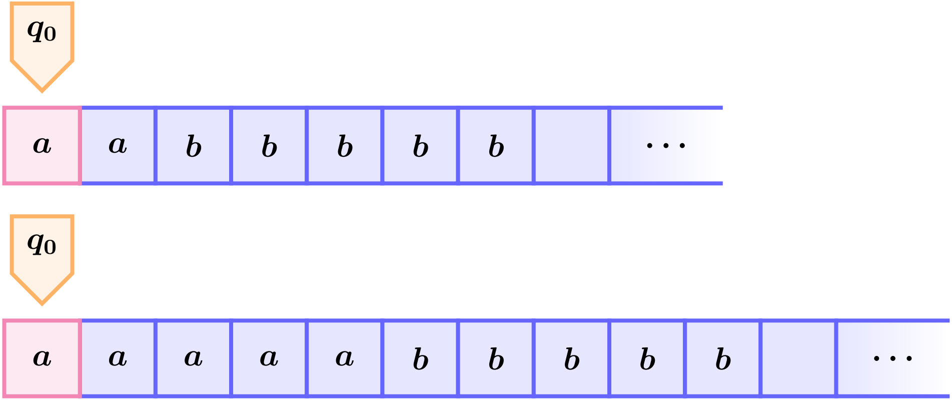
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\,b^k$ looks like:



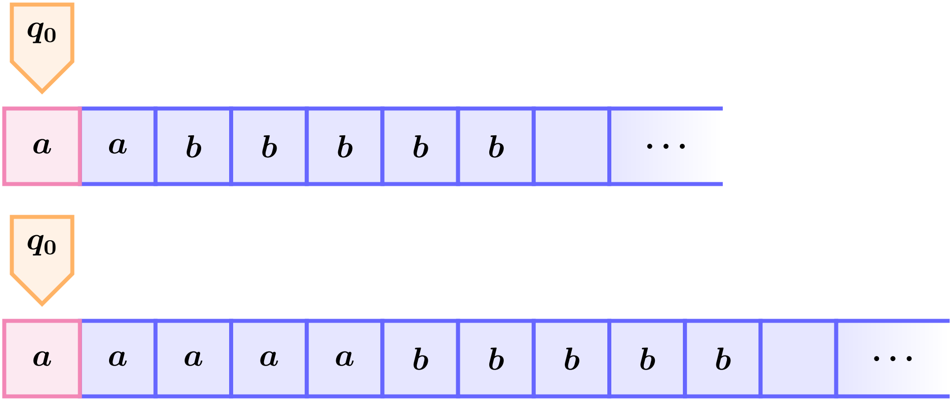
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



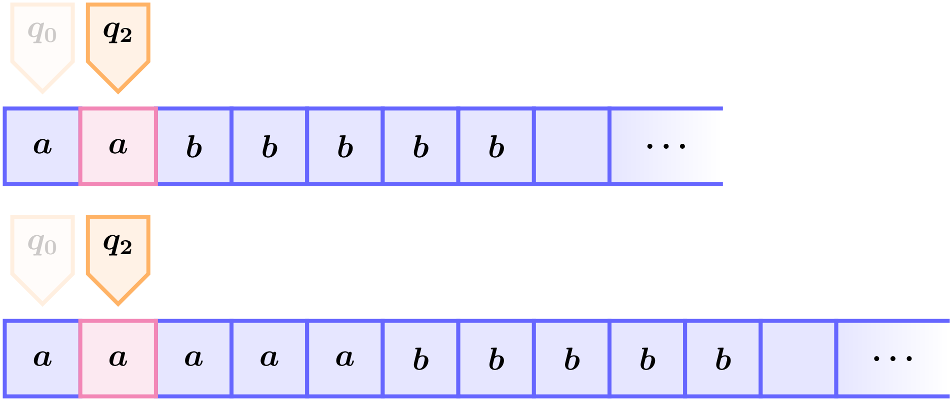
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\,b^k$ looks like:



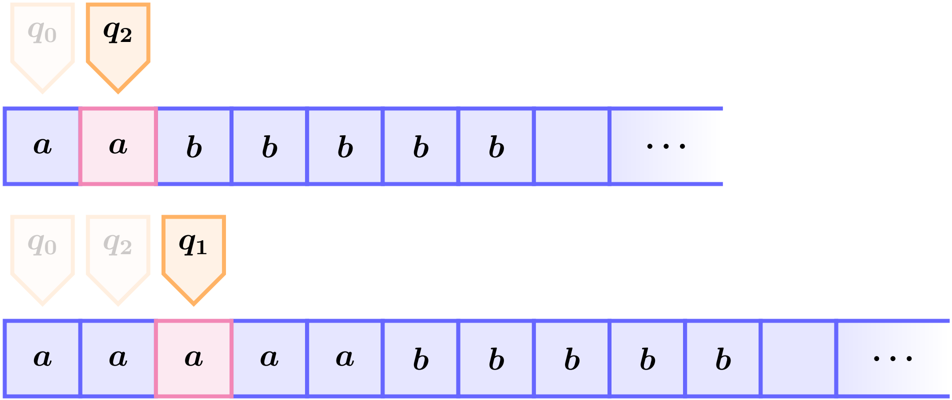
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\,b^k$ looks like:



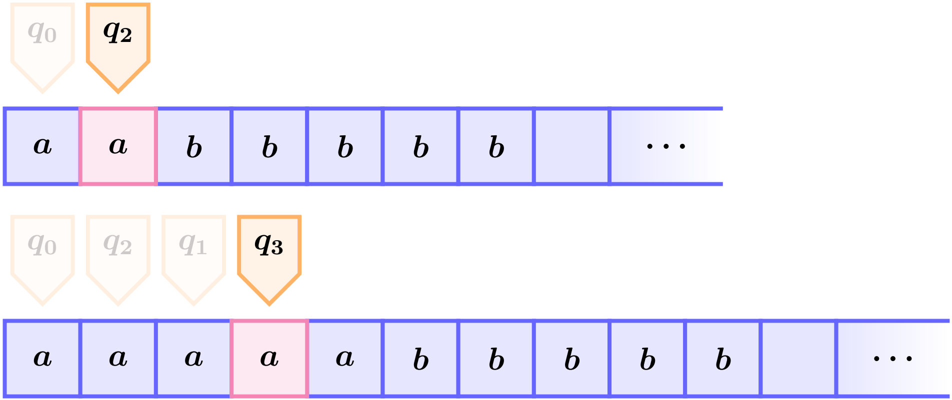
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



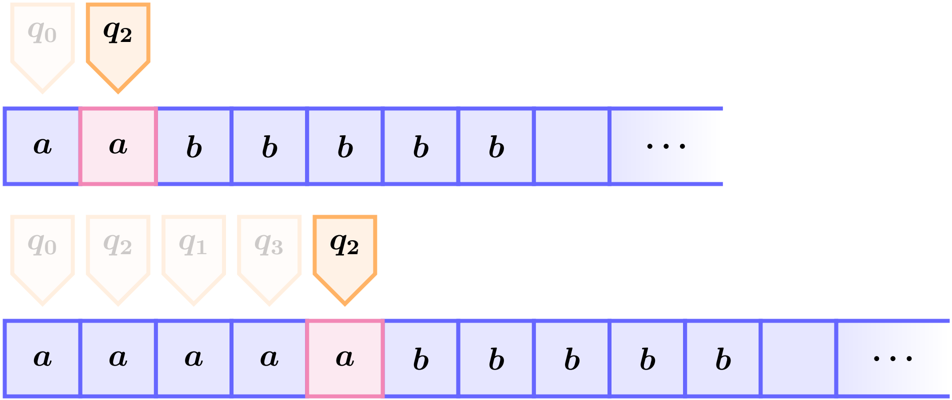
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



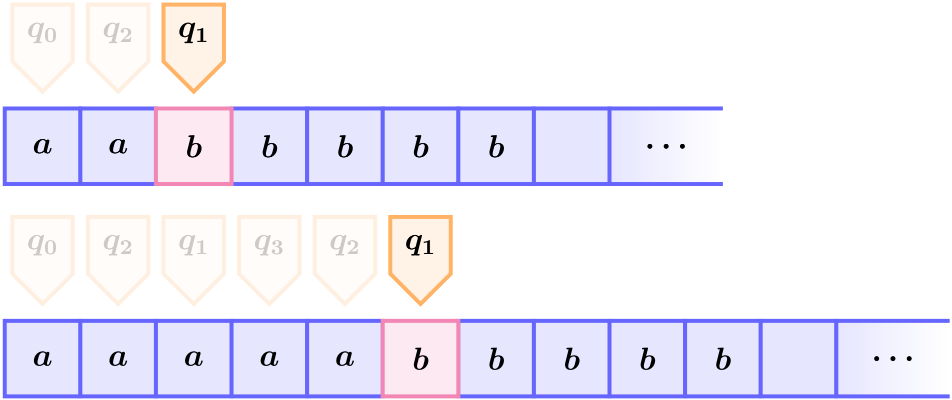
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



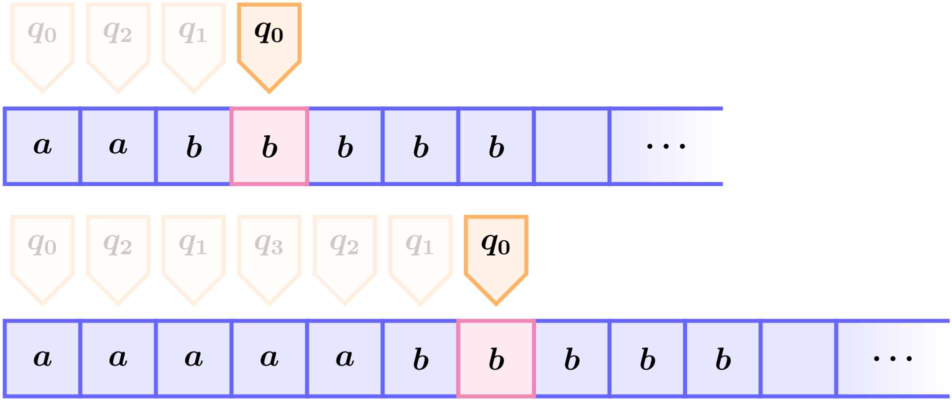
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



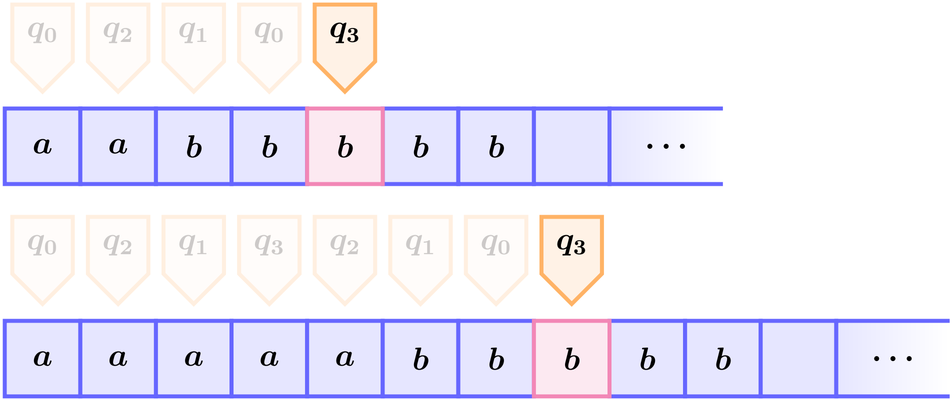
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



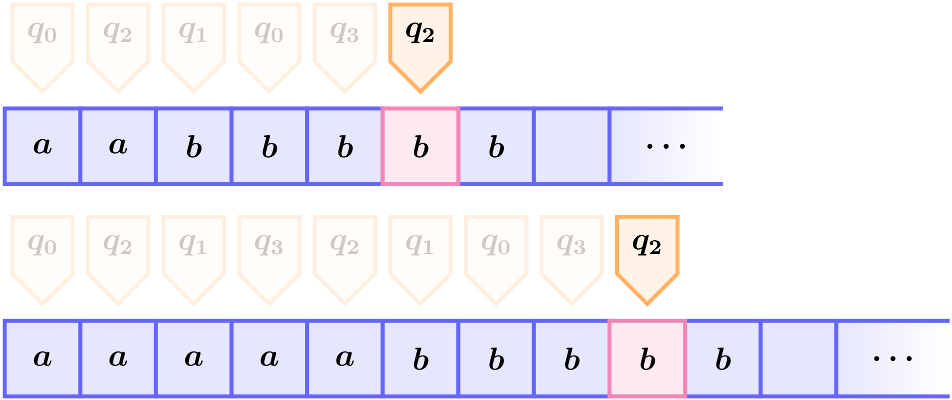
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



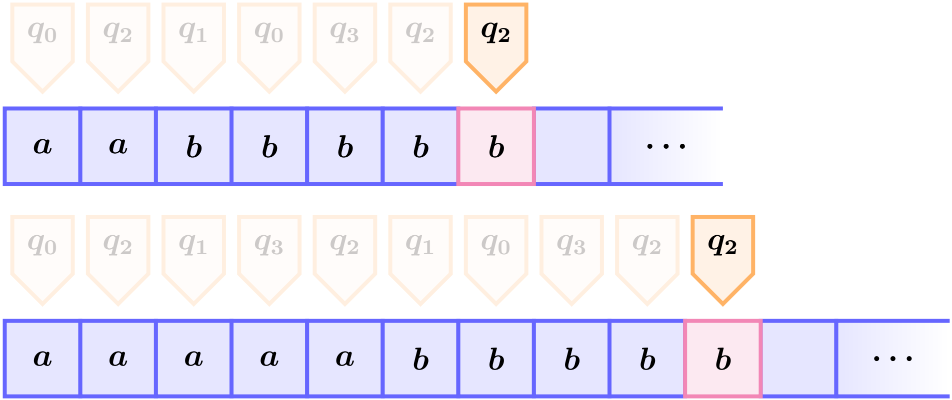
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



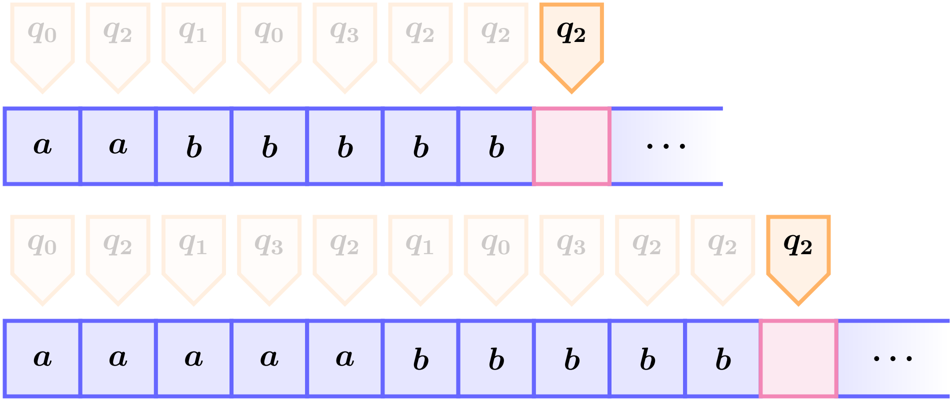
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



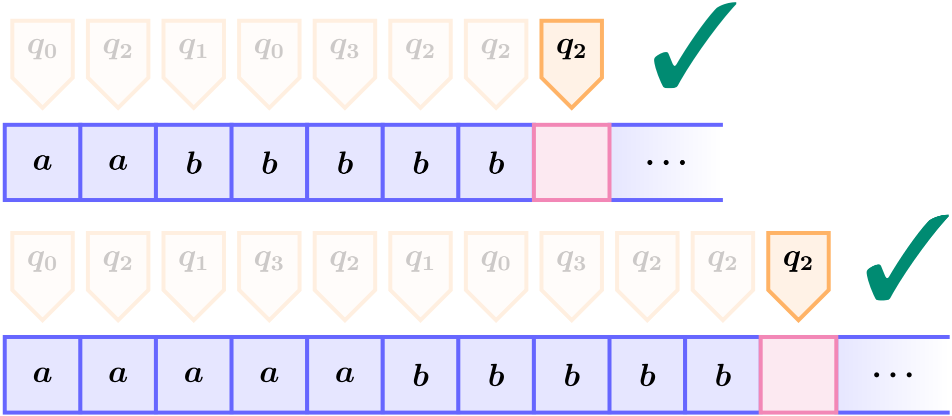
Not all languages are regular

Step 4) Consider what the computation of $a^{(k-\ell)}\, b^k$ looks like:



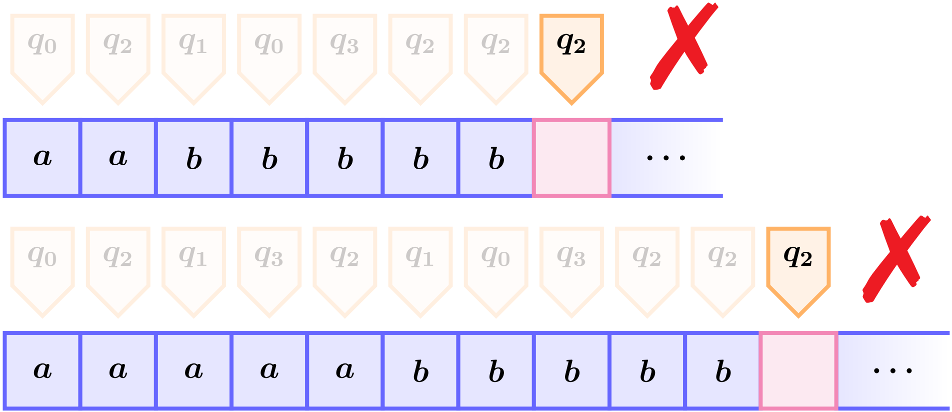
Not all languages are regular

Step 5) Either **both** computations **accept**



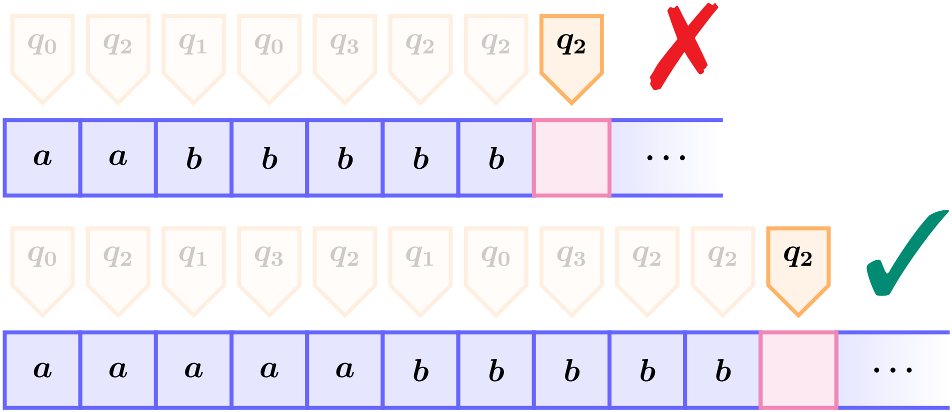
Not all languages are regular

Step 6) . . . Or **both** computations **reject**



Not all languages are regular

Step 7) But we assumed $A^\star$ recognised the language $\{a^n b^n : n = 0, 1, 2 \ldots\}$



Not all languages are regular

Step 8) Hence, by **contradiction**, there is no DFA that accepts the language

$$\boxed{\boldsymbol{L} = \{\boldsymbol{a^n b^n} : \boldsymbol{n} = 0, 1, 2, 3, \ldots\}}$$

Q.E.D.

The Pumping Lemma

The Pumping Lemma

Theorem (The Pumping Lemma) Let $L \subseteq \Sigma^*$ be an *infinite* language. If L is **regular** language then there is some $p > 0$ such that every word $w \in L$ of length $|w| \geq p$ can be written in the form $w = xyz$, for $x, y, z \in \Sigma^*$ such that:

- $|xy| \leq p$ and $|y| > 0$,
- $xy^n z \in L$ for all $n \geq 0$.

The Pumping Lemma

$$\boxed{L = \{a^n\, b^m \; : \; n < m\}}$$

End of Slides!