5CCS2FC2: Foundations of Computing II

Automata and Turing Machines

Week 1

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Deterministic Finite Automata

(a recap)

Languages

- **Alphabet / Words / Languages**
  - **Alphabet** A finite set of symbols. $\boxed{\Sigma = \{a_0, a_1, \ldots, a_k\}}$

Languages

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$$\boxed{\boldsymbol{\Sigma = \{a, b, c, \ldots, x, y, z\}}}$$

Languages

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  - **Alphabet** A finite set of symbols. $\boxed{\Sigma = \{0, 1\}}$

Languages

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$$\boxed{\boldsymbol{\Sigma} = \{\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \wedge, \vee, \rightarrow, \neg, (\ ,\ )\}}$$

Languages

- **Alphabet / Words / Languages**
  - **Alphabet** A finite set of symbols.

$\Sigma = \{$🧐, 🌍, 🎨, 😵‍💫, 🕴, 🐈$\}$

Languages

- **Alphabet / Words / Languages**
  - **Alphabet** A finite set of symbols. $\boxed{\Sigma = \{a_0, a_1, \ldots, a_k\}}$
  - **Words / Strings** $\boxed{\Sigma^* = \{ \text{ all finite strings from } \Sigma \}}$

    The set of all finite strings comprised of letters from the alphabet Σ.

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    The set of all finite strings comprised of letters from the alphabet Σ.
    - **The empty string** $\boxed{\varepsilon \in \Sigma^*}$

Languages

- Alphabet / Words / Languages

  - Languages $\boxed{L \subseteq \Sigma^*}$

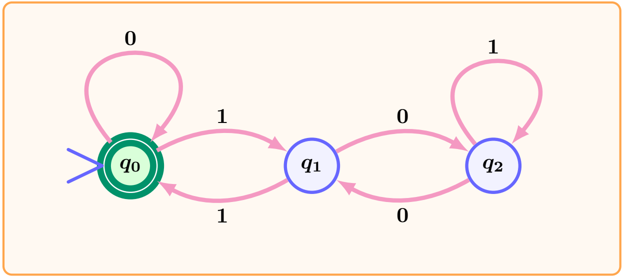
    Any subset (finite of infinite) of words is consider a language.

    - The empty language $\boxed{\emptyset \subseteq \Sigma^*}$

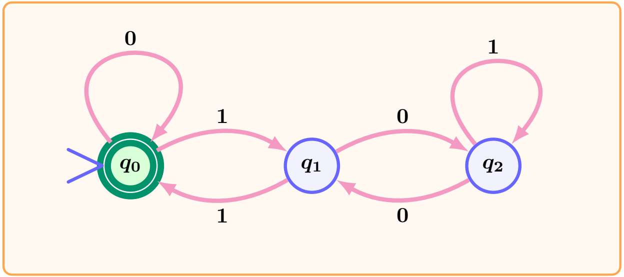
    - All English words

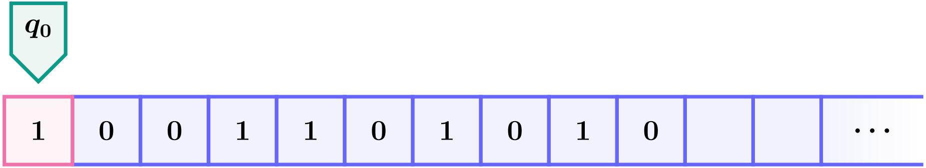
    - All binary strings representing prime numbers *etc.*

Deterministic Finite Automata (DFAs)



Deterministic Finite Automata (DFAs)





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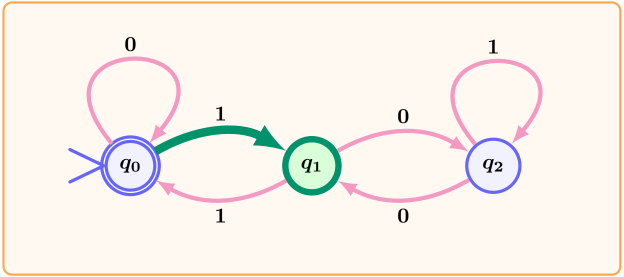

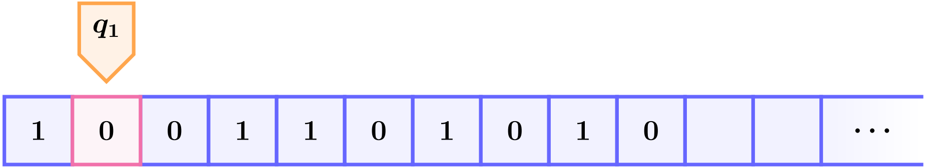

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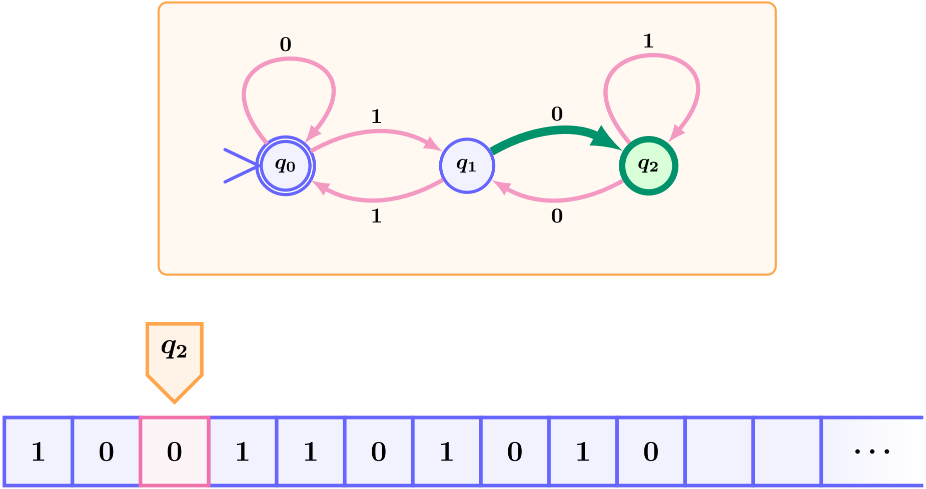

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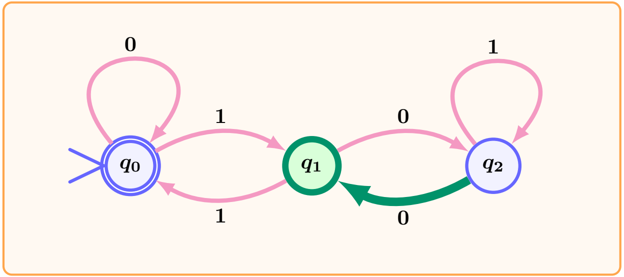

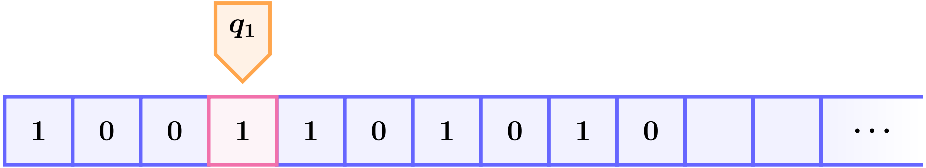

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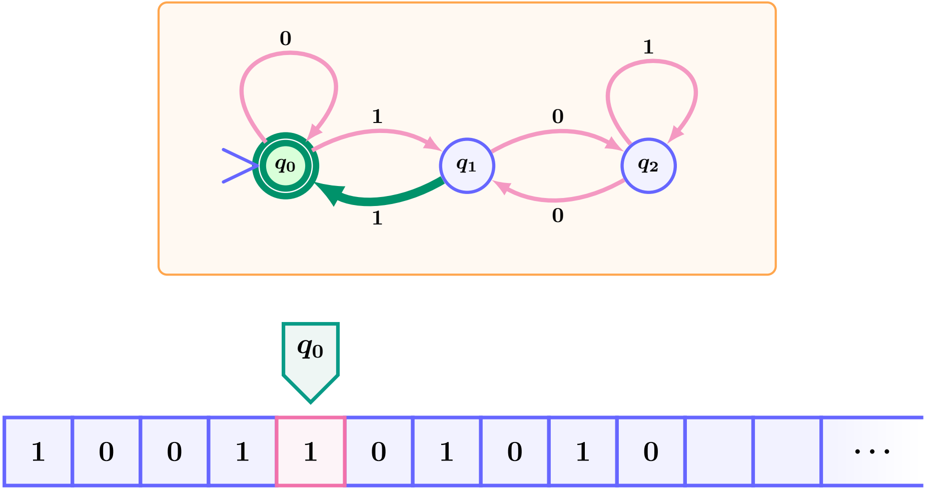

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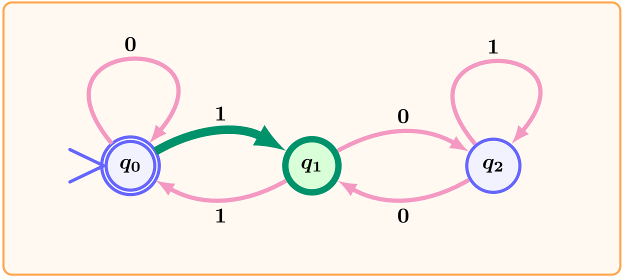

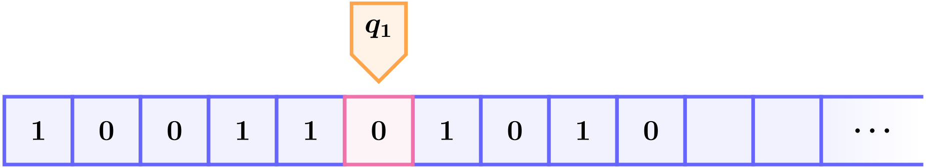

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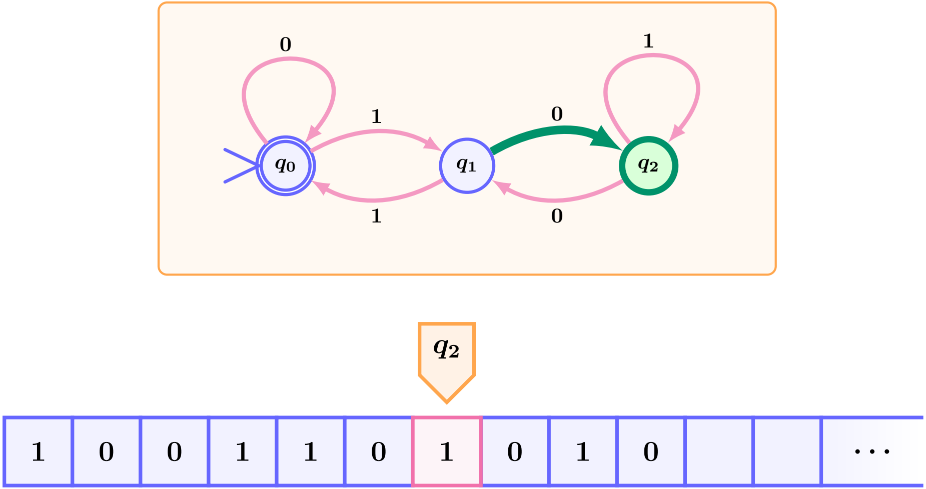

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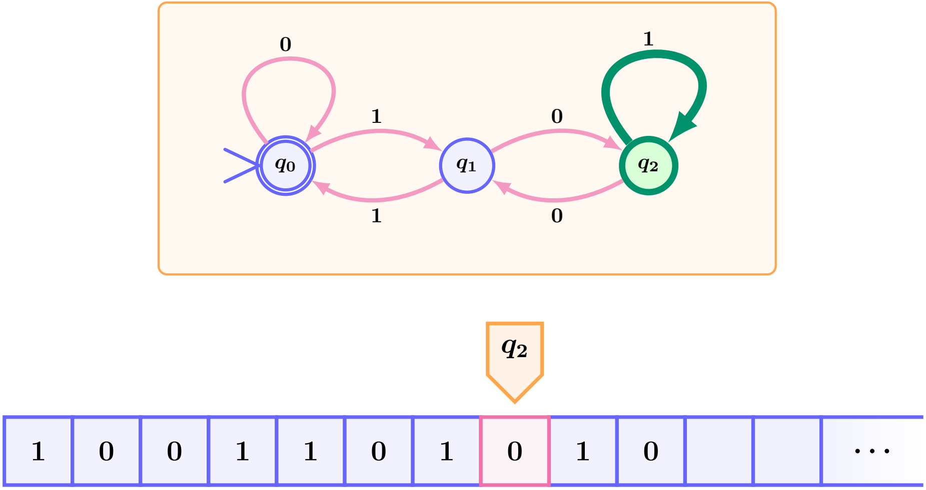

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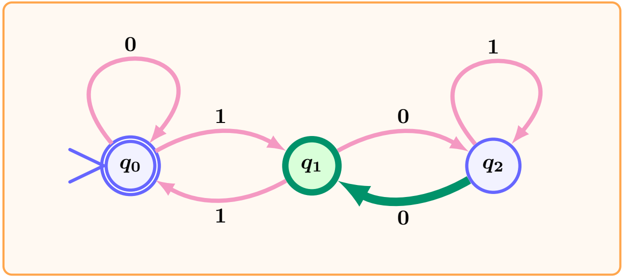

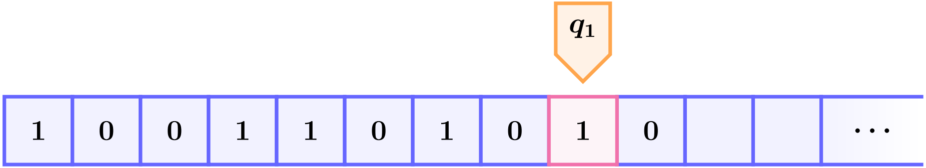

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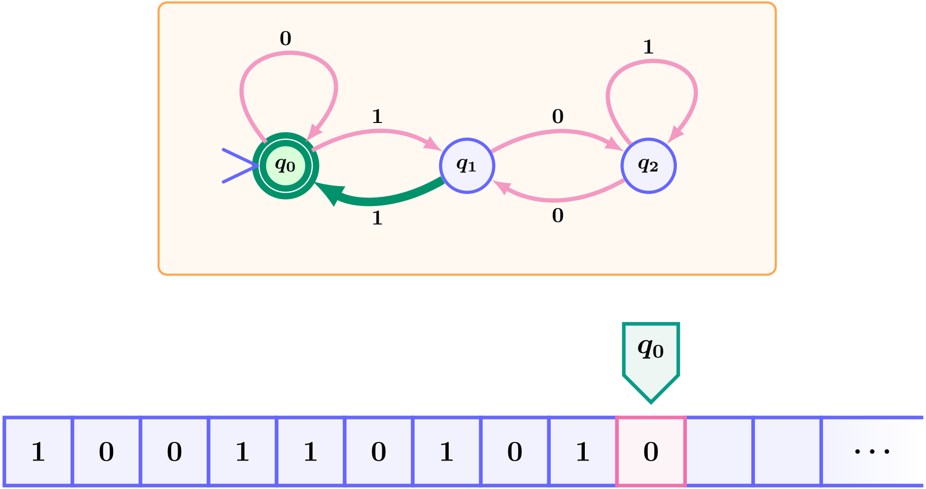

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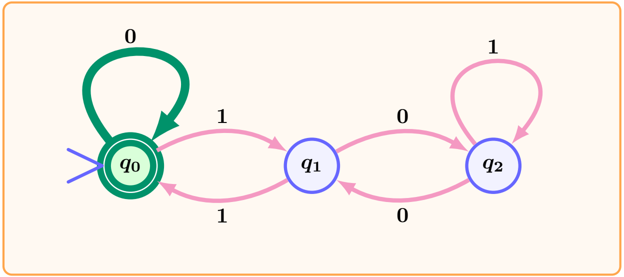

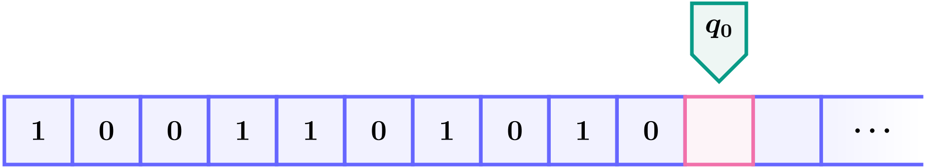

Deterministic Finite Automata (DFAs)

- **Deterministic Finite Automata**

$$\boxed{A \;=\; \langle\, \Sigma,\; Q,\; q_{\mathsf{init}},\; F,\; \delta \,\rangle}$$

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  - Finite set of control states

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$$\boxed{A \;=\; \langle\, \Sigma,\; Q,\; q_{\mathsf{init}},\; F,\; \delta \,\rangle}$$

  - **Transition function**

$$\boxed{\delta : Q \times \Sigma \to Q}$$

$$\boxed{\delta\,\big(\,\text{current state}\;,\;\text{read symbol}\,\big)\;\mapsto\;\text{new state}}$$

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Current State	Read Symbol	New State
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q_0	1	q_1
q_1	0	q_2
q_1	1	q_0
q_2	0	q_1
q_2	1	q_2

Deterministic Finite Automata (DFAs)

- **Configuration** A complete description of the machine at any given time

$$\boxed{(\text{ current state, substring to the right of tape head })}$$

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- **Configuration** A complete description of the machine at any given time

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- **Computation** The sequence of configurations obtained by running the DFA
  - **Example:**

$$\boxed{(q_0, 1001) \rightarrow (q_1, 001) \rightarrow (q_2, 01) \rightarrow (q_1, 1) \rightarrow (q_0, \varepsilon)}$$

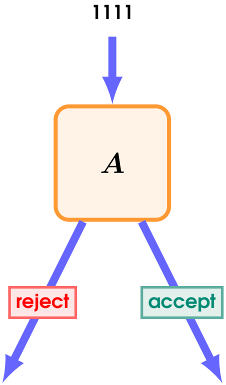
Deterministic Finite Automata (DFAs)

- **Accept Criterion**

A accepts w	$\iff$	The final state of the (unique) computation starting (q_{init}, w) is an accepting state

Deterministic Finite Automata (DFAs)

- **Decision Procedure**



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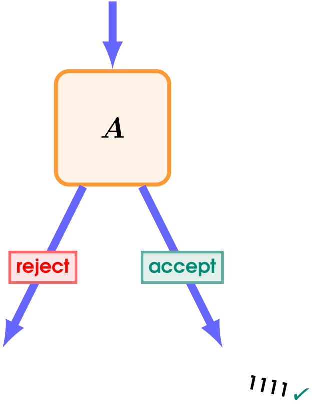

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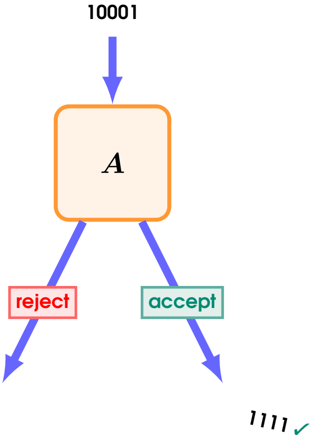

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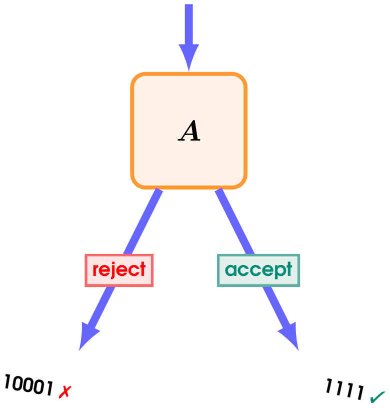

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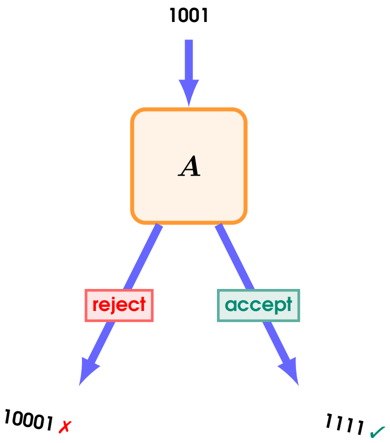

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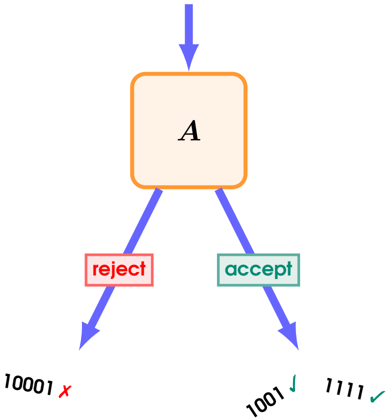

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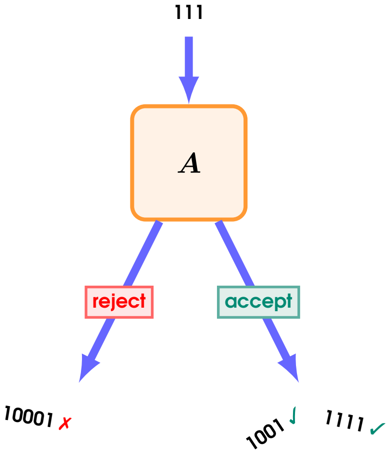

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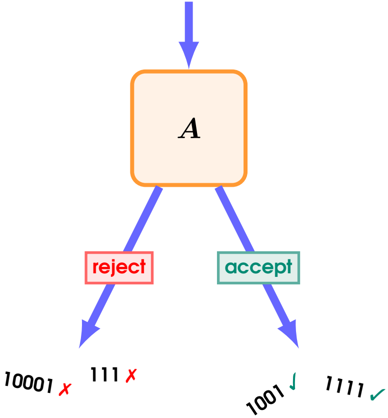

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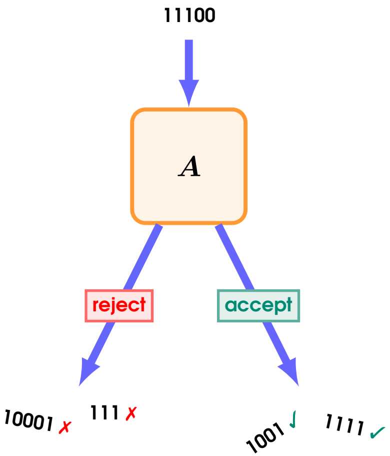

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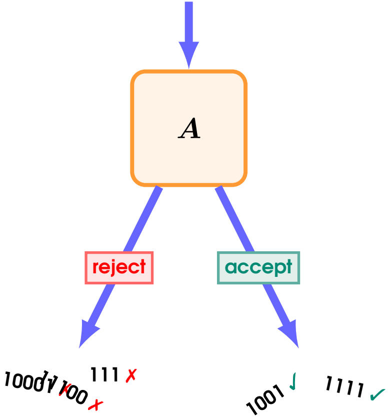

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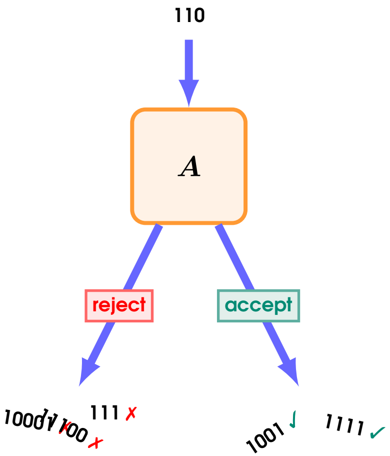

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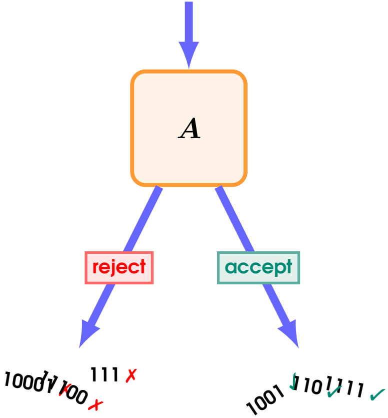

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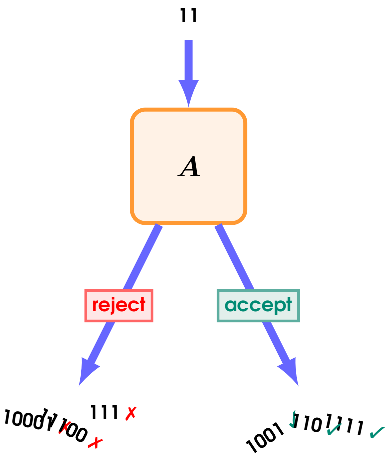

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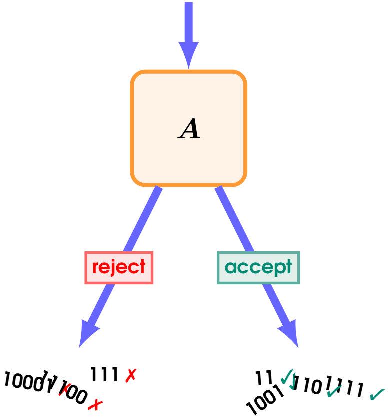

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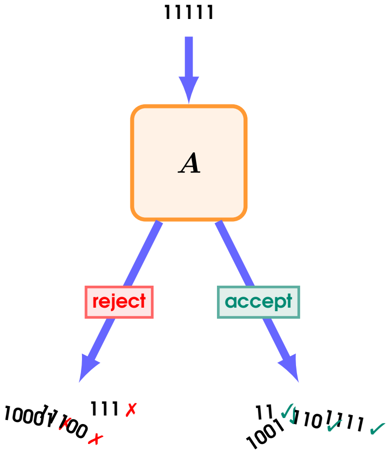

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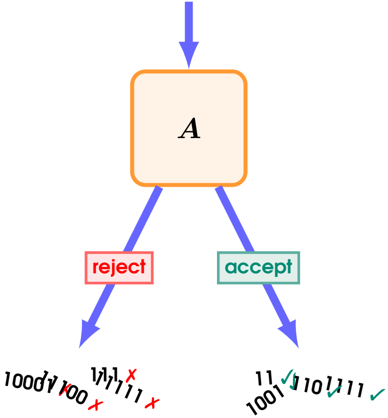

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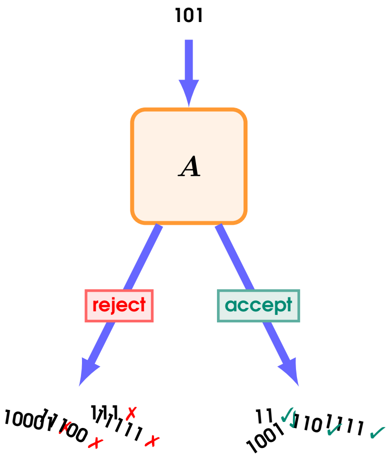

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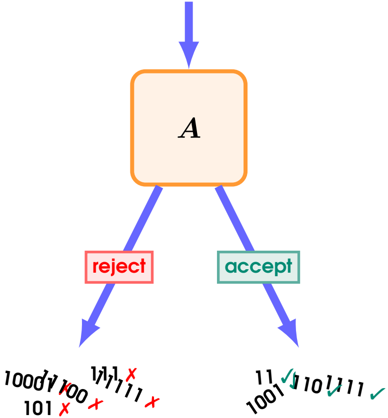

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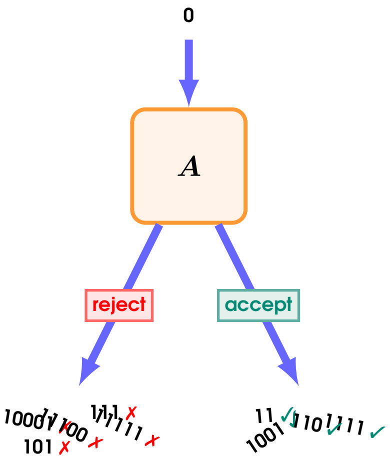

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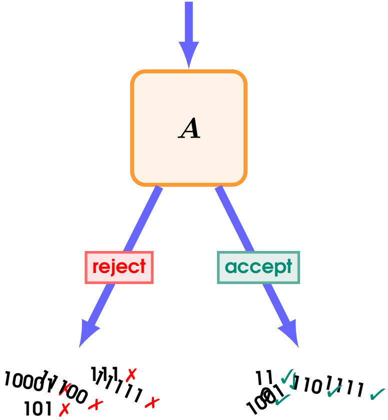

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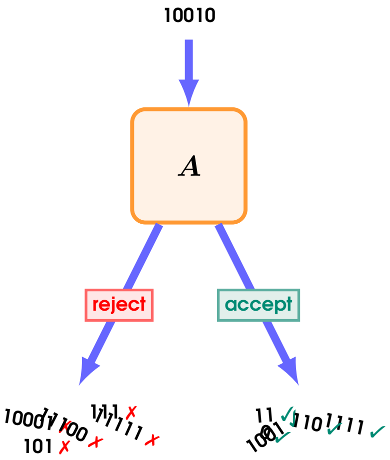

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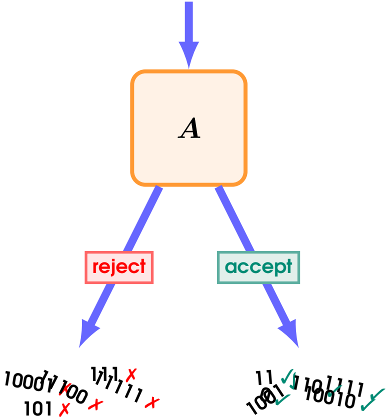

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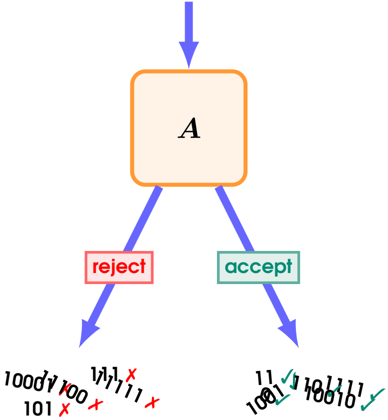

$$\textbf{Language}\,(A) \;=\; \{\; w \in \Sigma^* \;:\; A \text{ accepts } w \;\}$$

Non-deterministic Finite Automata

(a recap)

Non-deterministic Finite Automata (NFAs)

- Non-deterministic Finite Automata $\boxed{A = \langle \Sigma, Q, q_{\mathsf{init}}, F, \delta \rangle}$

  - Alphabet $\boxed{\Sigma = \{a_1, \ldots, a_k\}}$
  - Finite set of control states $\boxed{Q = \{q_1, \ldots, q_n\}}$
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As before!

Non-deterministic Finite Automata (NFAs)

- **Non-deterministic Finite Automata**

$$\boxed{A = \langle\, \Sigma,\ Q,\ q_{\text{init}},\ F,\ \delta\,\rangle}$$

  - **Transition function**

$$\boxed{\delta : Q \times \Sigma \to \mathcal{P}(Q)}$$

(where $\mathcal{P}(Q)$ denotes the powerset of Q)

$$\boxed{\delta\left(\text{current state}\ ,\ \text{read symbol}\right) \mapsto \left\{\text{all possible new states}\right\}}$$

Non-deterministic Finite Automata (NFAs)

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Current State	Read Symbol	New State
s	a	$\{q\}$
s	b	$\{q, f\}$
q	a	$\{f\}$
q	b	$\emptyset$
f	a	$\{f\}$
f	b	$\{s, f\}$

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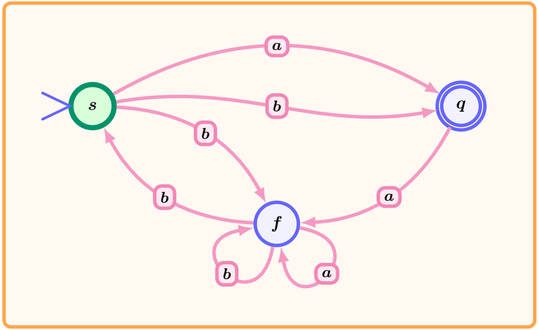

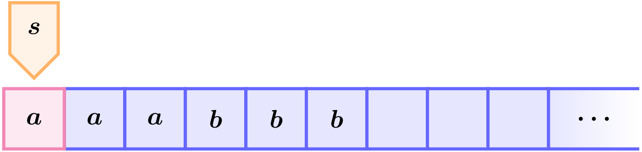

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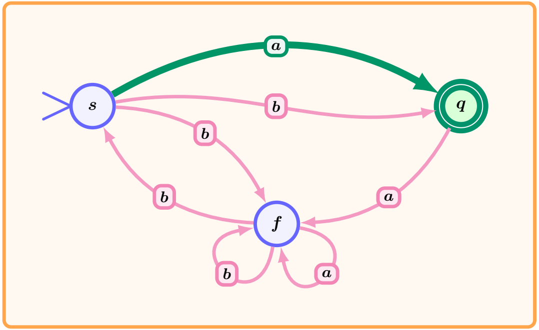

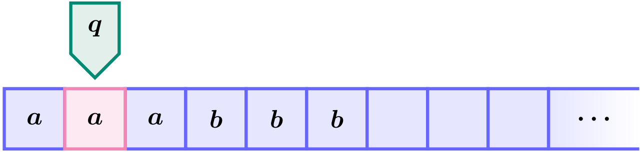

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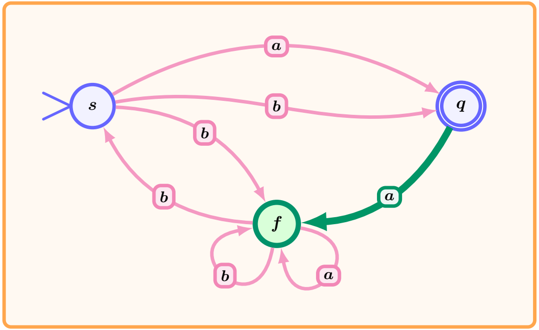

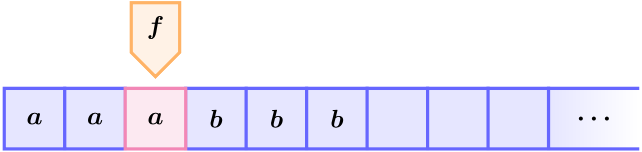

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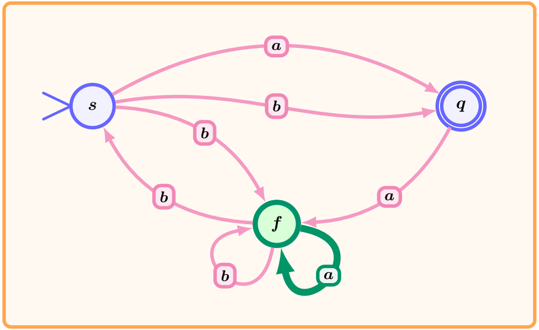

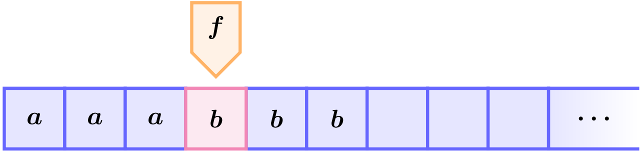

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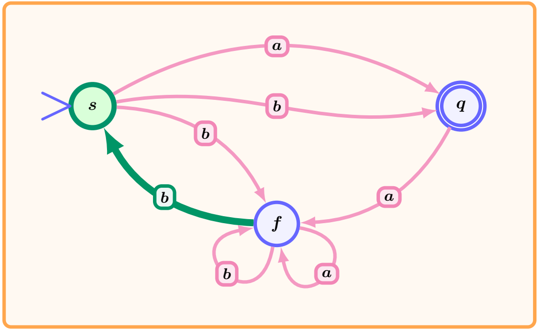

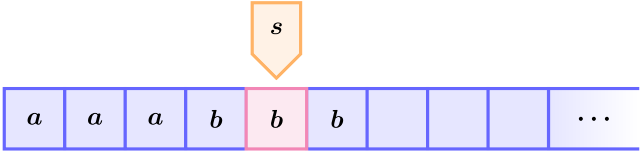

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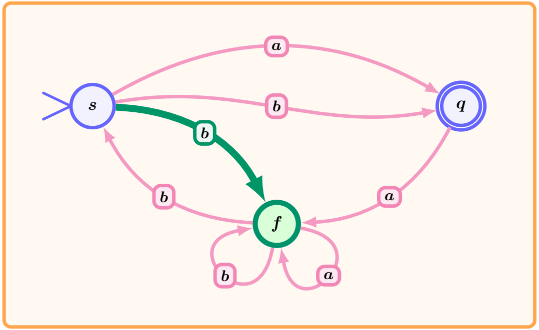

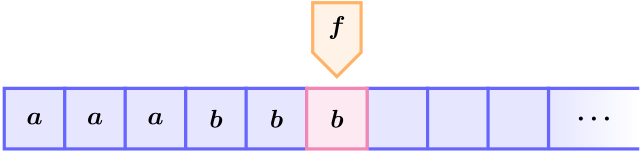

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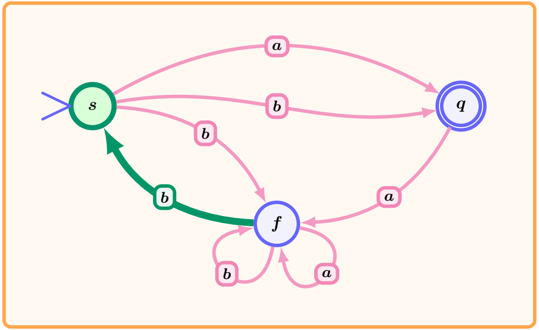

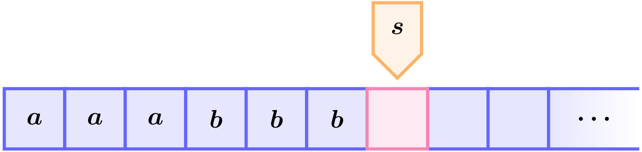

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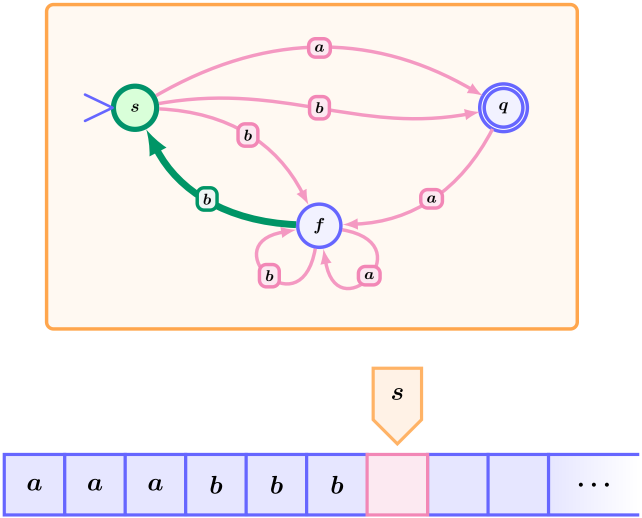

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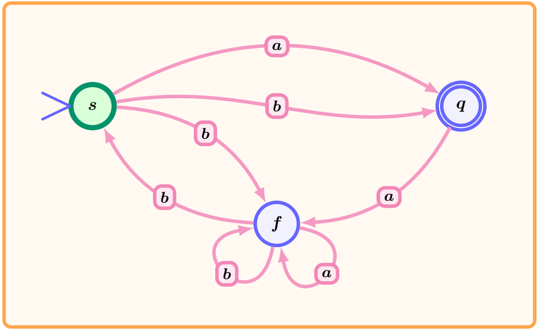

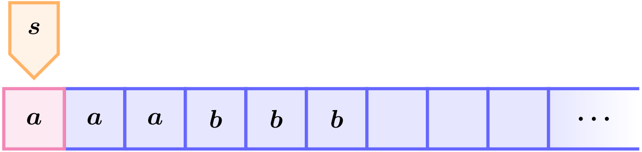

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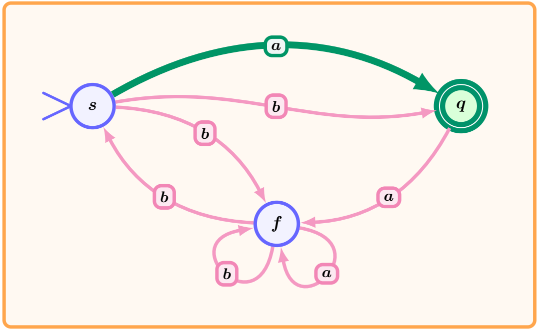

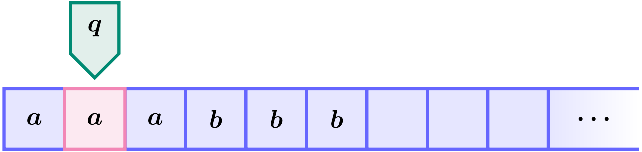

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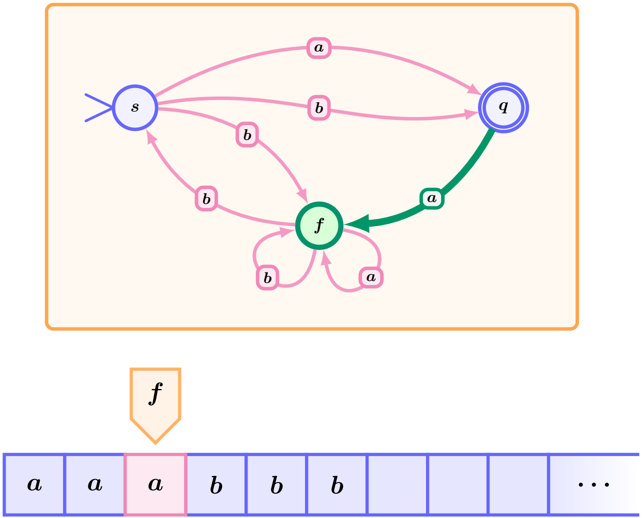

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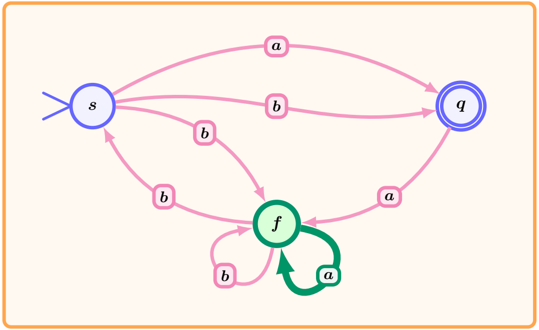

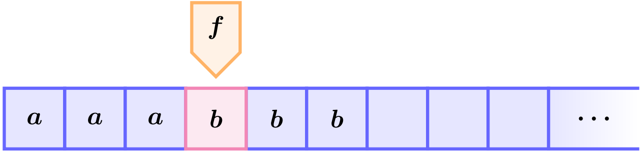

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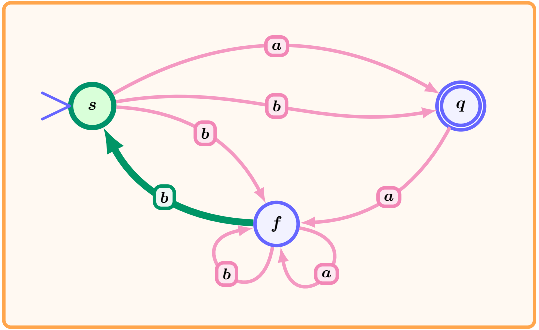

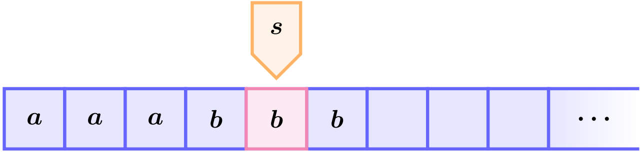

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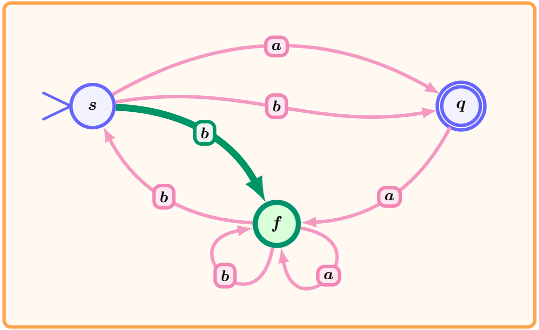

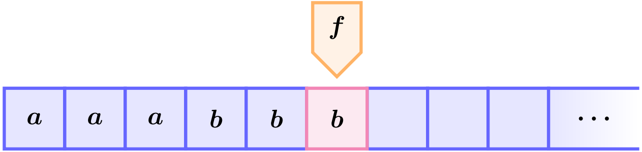

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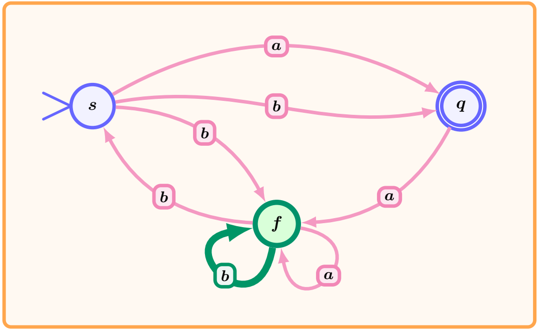

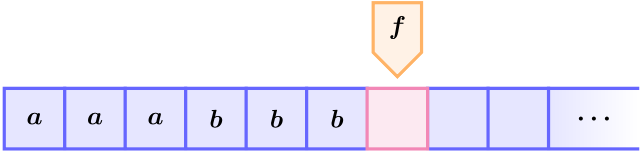

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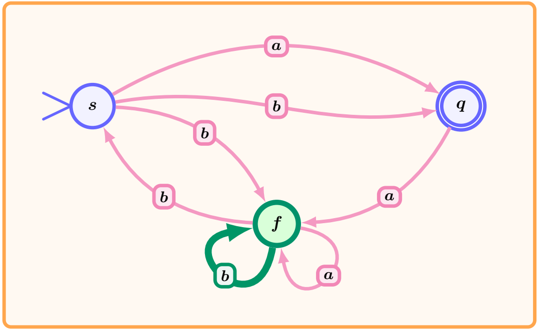

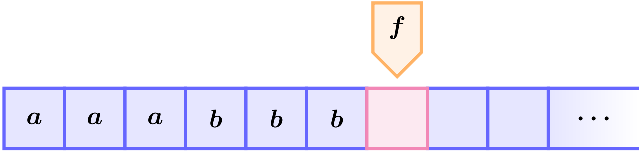

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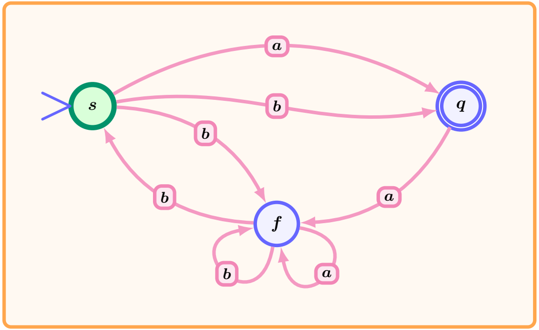

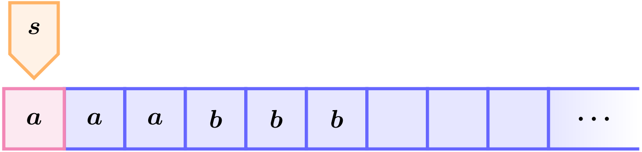

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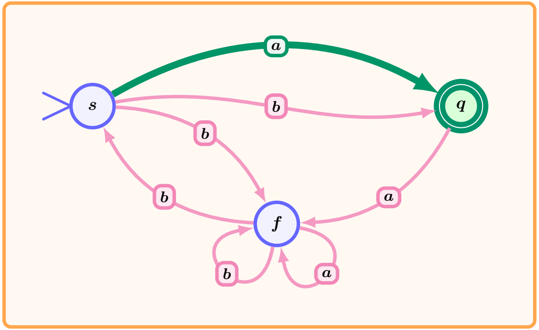

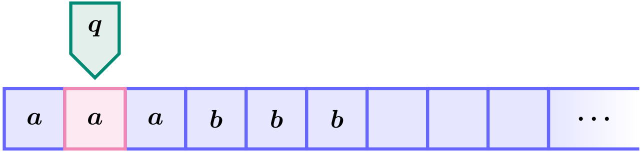

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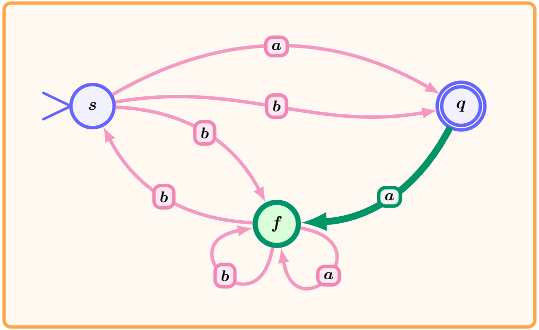

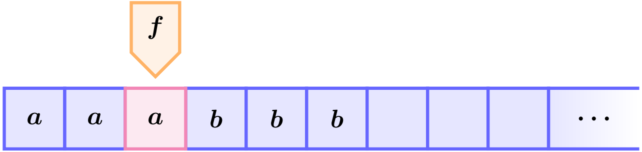

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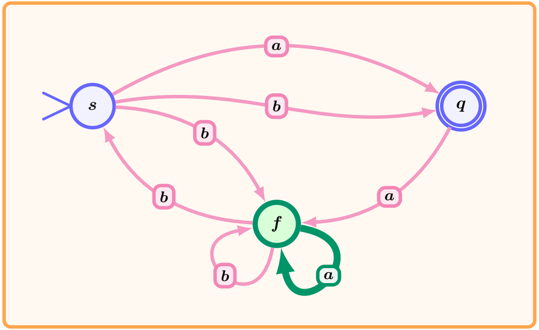

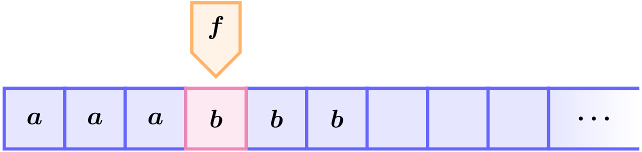

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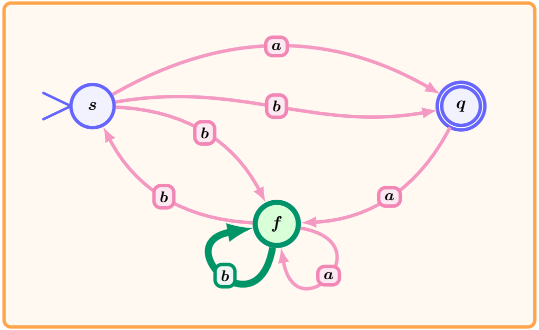

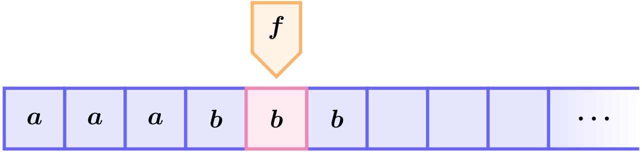

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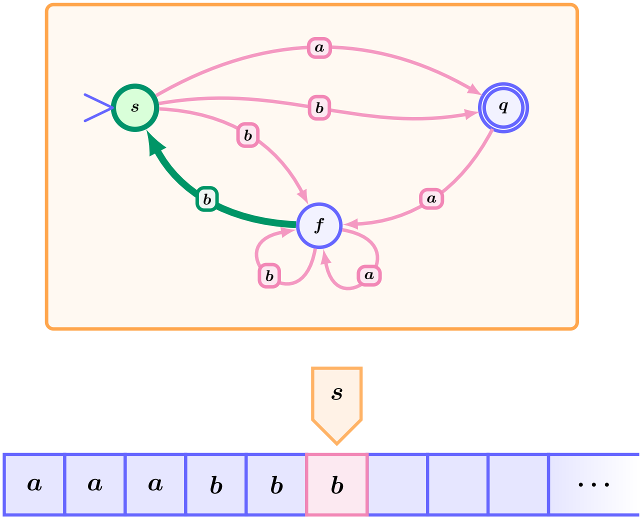

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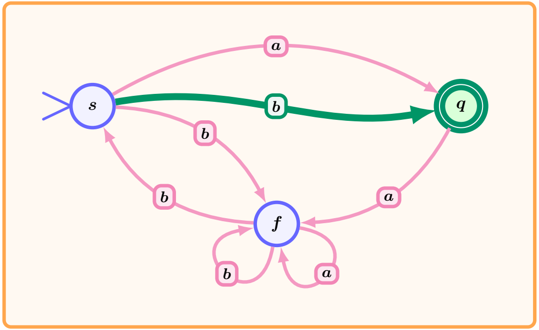

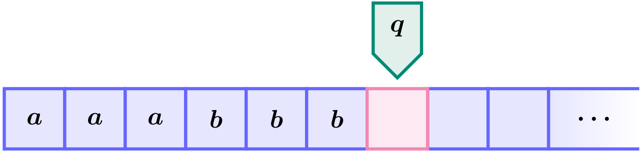

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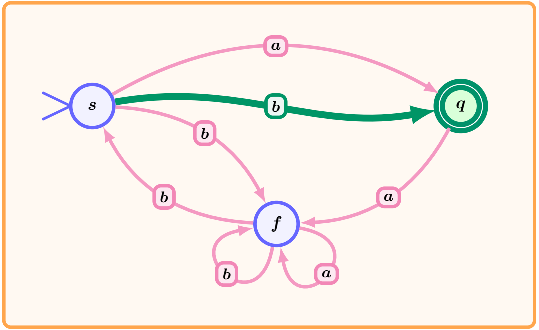
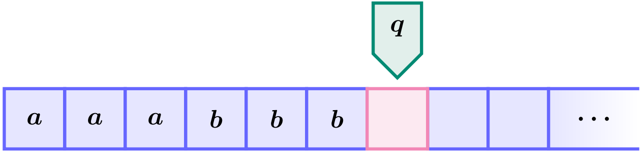
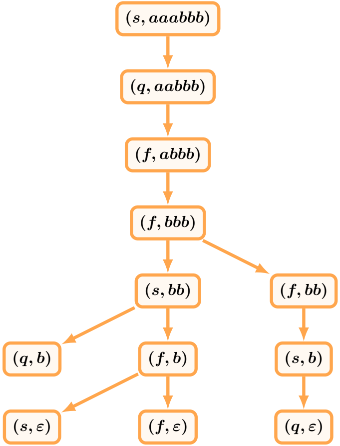




Non-deterministic Finite Automata (NFAs)

- Computation



Non-deterministic Finite Automata (NFAs)

- **Accept Criterion for NFAs**

A accepts w	$\iff$	The final state of the **ANY** computation starting (q_{init}, w) is an accepting state

Non-deterministic Finite Automata (NFAs)

> **Theorem** Every NFA can be **converted** to a DFA that accepts the same language.

Proof:

- Recall the **power-set construction** from FC1.

Q.E.D.

Regular Expressions and Regular Languages

Regular Expressions and Regular Languages

- **Regular Expression** Any word that can be constructed using only the following rules:
  - **Concatenation** $\boxed{wv}$
  - **Alternation / Choice** $\boxed{(w \cup v)}$
  - **Kleene Star** $\boxed{w^*}$

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  - **Kleene Star** $\boxed{w^*}$
- **Regular Language** Any set of words that can be 'represented' by a regular expression.

Regular Expressions and Regular Languages

- **Some Examples:**
  - All binary strings that start with a one and end with a zero,

$$\boxed{1(1 \cup 0)^*0}$$

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  - All strings of **even length** containing only $\square$ and $\diamond$,

$$\boxed{((\square \cup \diamond)(\square \cup \diamond))^*}$$

Regular Expressions and Regular Languages

- **Some Examples:**
  - **Less Obviously:** All binary strings representing multiples of three,

$$\boxed{\left(0 \cup 1(01^*0)^*1\right)^*}$$

Regular $\Longrightarrow$ Recognised by some DFA/NFA

Regular $\Leftarrow$ **Recognised by some DFA/NFA**

Regular $\Leftrightarrow$ Recognised by some DFA/NFA

Are all languages regular?

End of Slides!