5CCS2FC2: Foundations of Computing II

Automata and Turing Machines

Week 1

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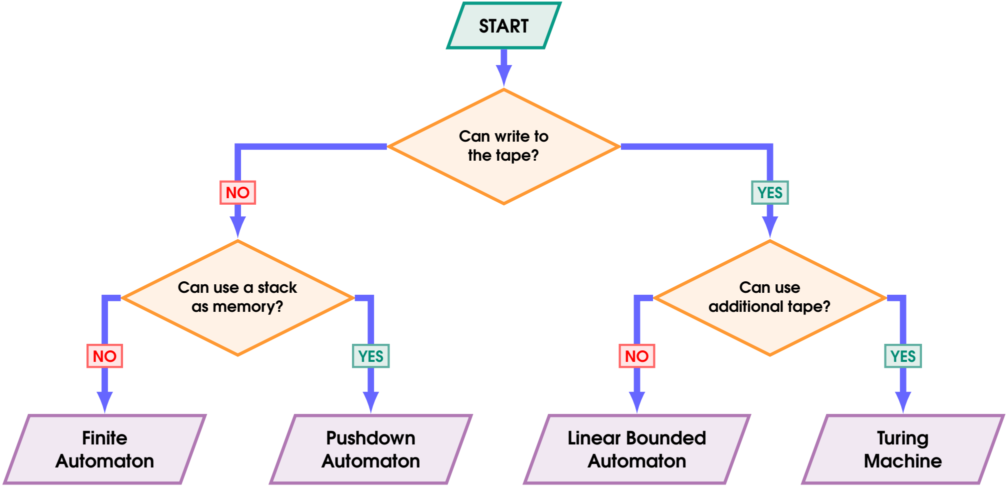
Turing Machines

Turing Machines



Turing Machines

- **Turing Machine** $\boxed{M = \langle\, \Sigma,\ Q,\ q_{\text{init}},\ q_{\text{accept}},\ q_{\text{reject}},\ \delta \,\rangle}$

  - Alphabet $\boxed{\Sigma = \{a_1, \ldots, a_k\}}$
  - Finite set of control states $\boxed{Q = \{q_1, \ldots, q_n\}}$
  - Initial state $\boxed{q_{\text{init}} \in Q}$
  - Accept and Reject states $\boxed{q_{\text{accept}}, q_{\text{reject}} \in Q}$

As before!

Turing Machines

- **Turing Machine**

$$\boxed{M \;=\; \langle\, \Sigma,\; Q,\; q_{\text{init}},\; q_{\text{accept}},\; q_{\text{reject}},\; \delta \,\rangle}$$

  - **Transition function**

$$\boxed{\delta : (Q \times \Sigma \cup \{\sqcup\}) \rightarrow (Q \times \Sigma \cup \{\sqcup\} \times \{\leftarrow, \rightarrow\})}$$

$$\boxed{\delta \;(\text{ current state , read symbol }) \;\mapsto\; (\text{ new state, write, move })}$$

Turing Machines

- **Turing Machine**

$$M = \langle \Sigma, Q, q_{\text{init}}, q_{\text{accept}}, q_{\text{reject}}, \delta \rangle$$

- **Transition function**

$$\delta : (Q \times \Sigma \cup \{\sqcup\}) \rightarrow (Q \times \Sigma \cup \{\sqcup\} \times \{\leftarrow, \rightarrow\})$$

Current State	Read Symbol	New State	Write Symbol	Move Head
q_{init}	a	q_1	$\sqcup$	$\rightarrow$
q_{init}	$\sqcup$	q_{accept}	$\sqcup$	$\rightarrow$
q_1	a	q_1	a	$\rightarrow$
q_1	b	q_1	b	$\rightarrow$
q_1	$\sqcup$	q_2	$\sqcup$	$\leftarrow$
q_2	b	q_3	$\sqcup$	$\leftarrow$
q_3	a	q_3	a	$\leftarrow$
q_3	b	q_3	b	$\leftarrow$
q_3	$\sqcup$	q_{init}	$\sqcup$	$\rightarrow$

Turing Machines

- **Configuration** A complete description of the machine at any given time

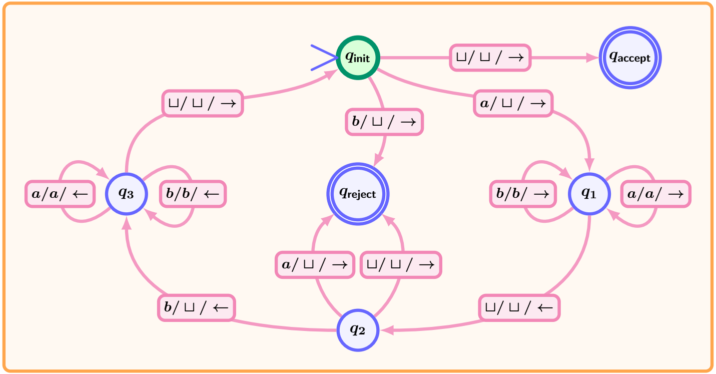
$$\boxed{(\text{ current state, left substring, right substring })}$$

- **Computation** The sequence of configurations obtained by running the TM

- **Accept Criterion**

M accepts w	$\Longleftrightarrow$	M terminates and the final state of the computation starting $(q_{\text{init}}, \varepsilon, w)$ is the accepting state q_{accept}

Turing Machines

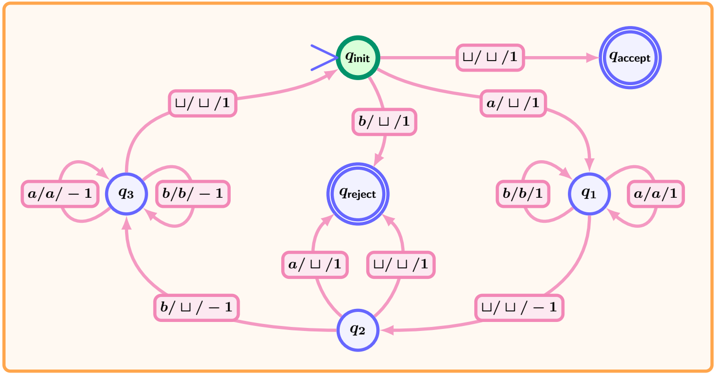


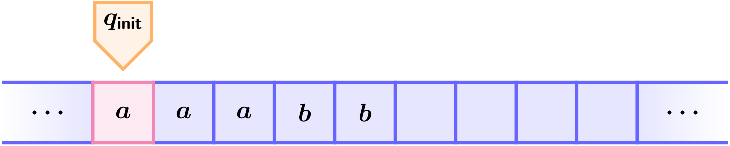
q_{init}

Accept ✓

$\cdots$	a	a	a	b	b	b				$\cdots$

Turing Machines





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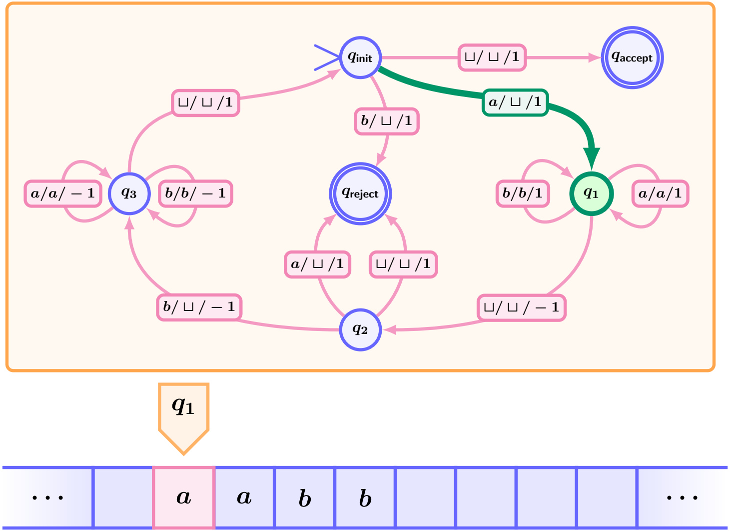

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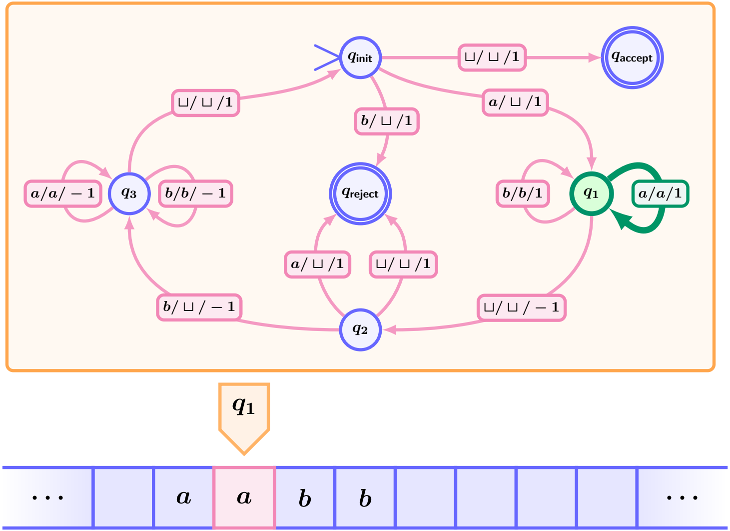

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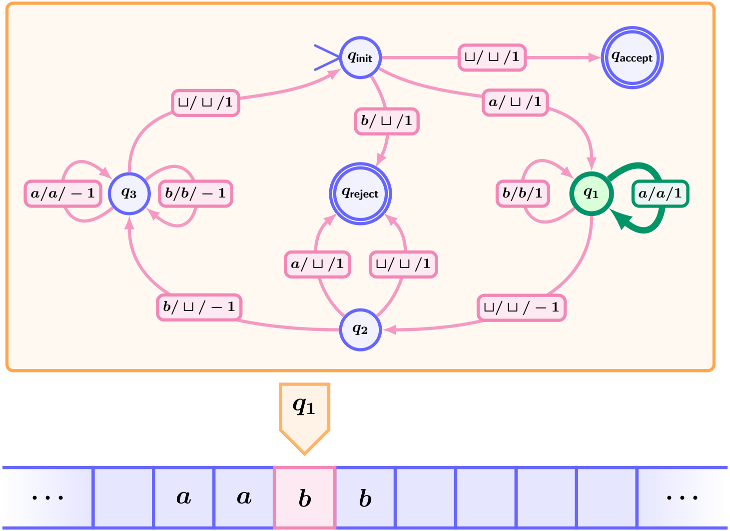

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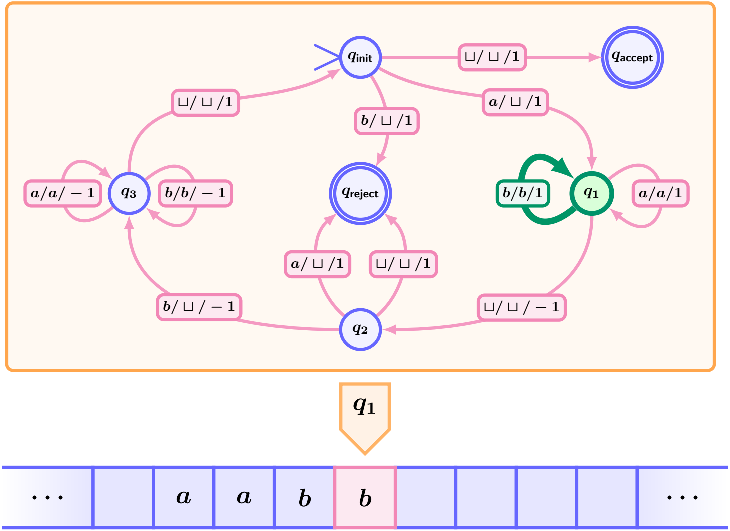

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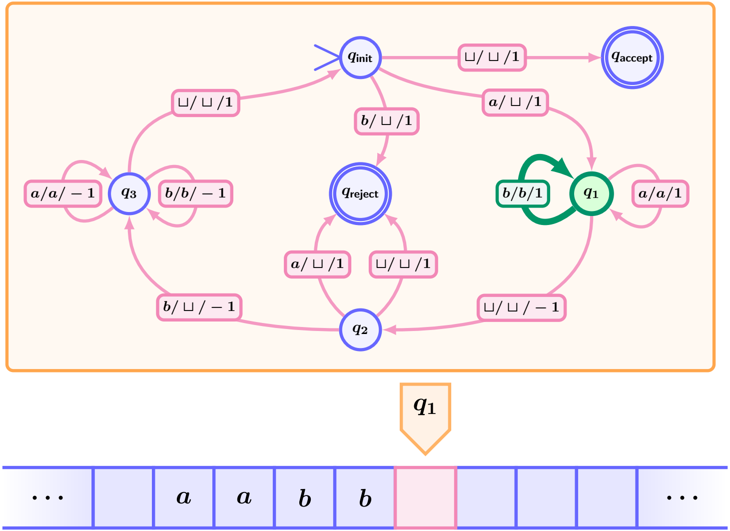

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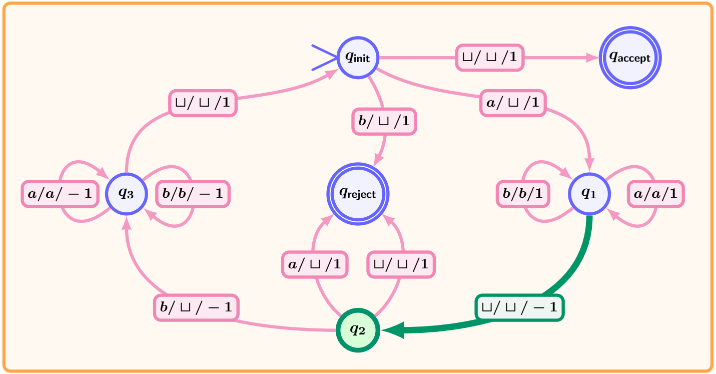

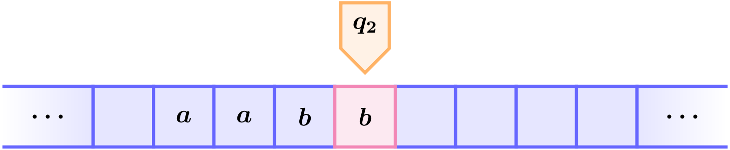

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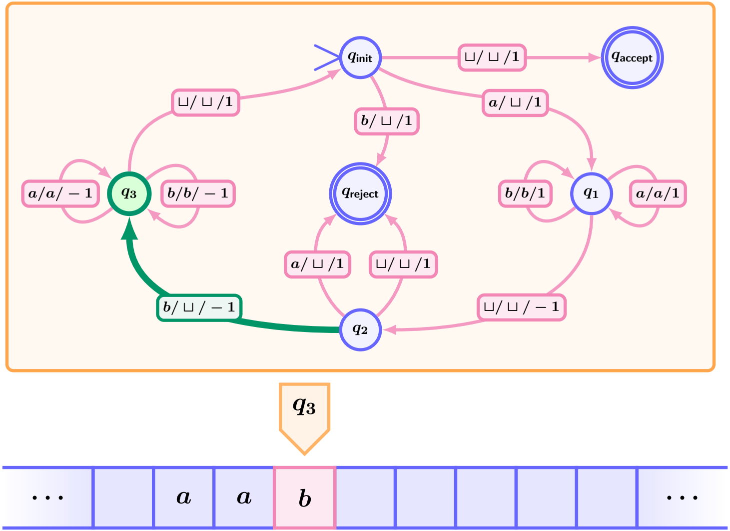

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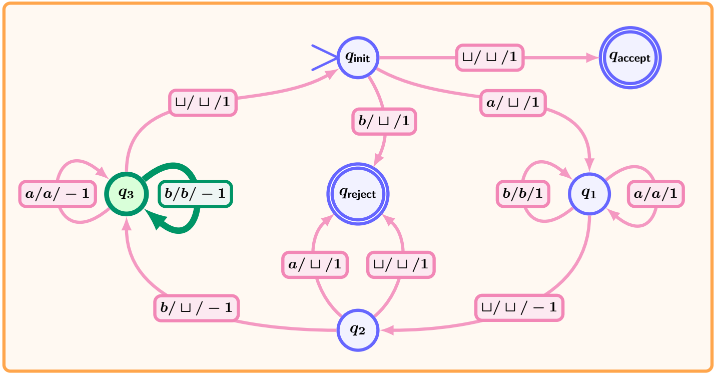

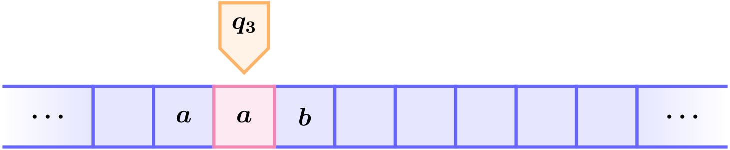

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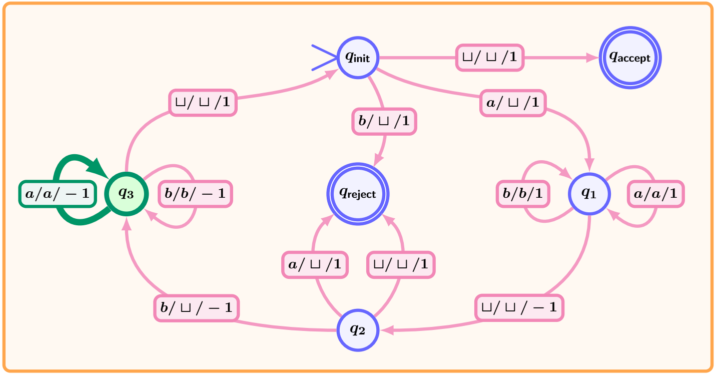

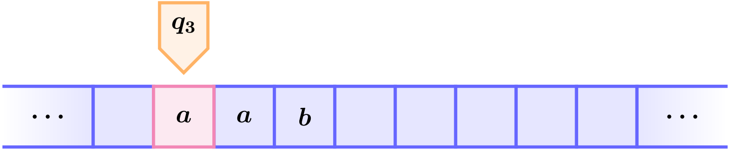

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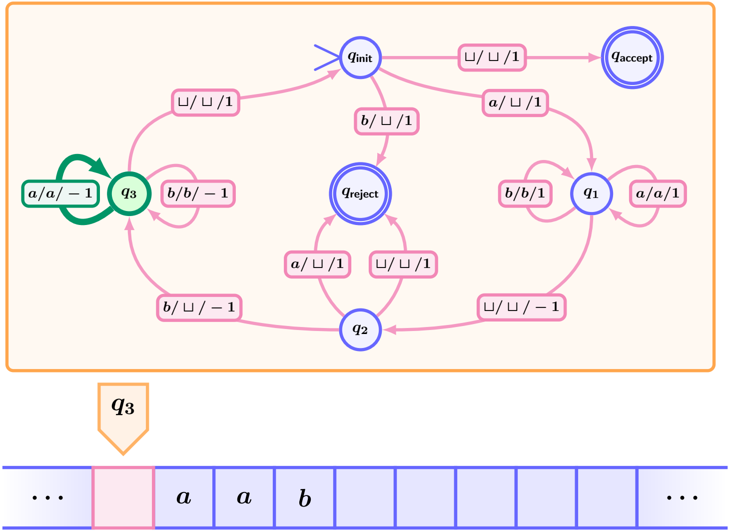

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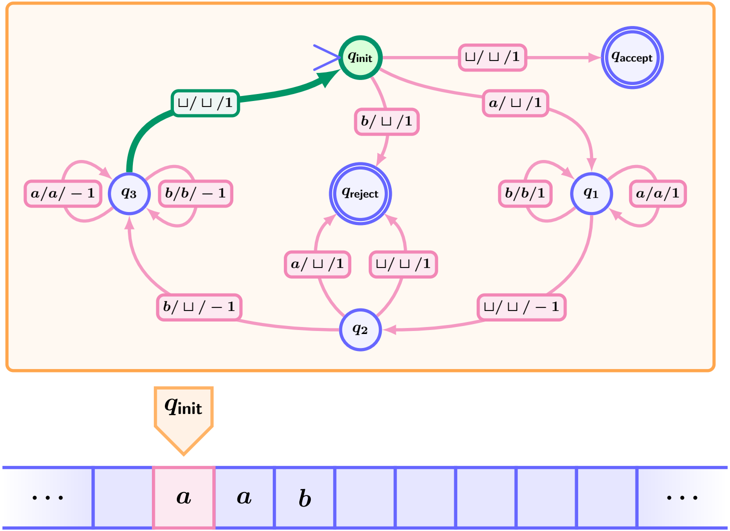

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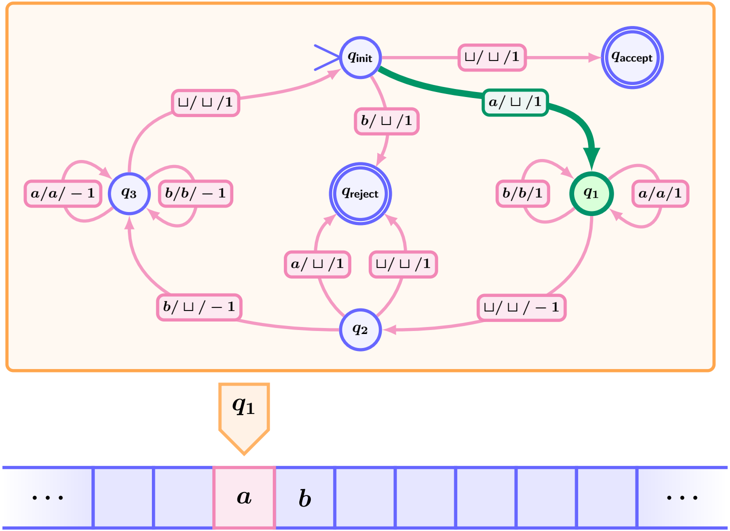

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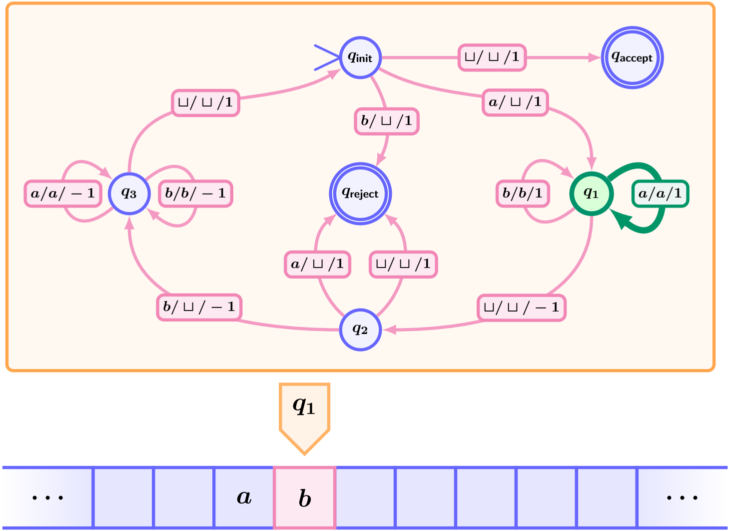

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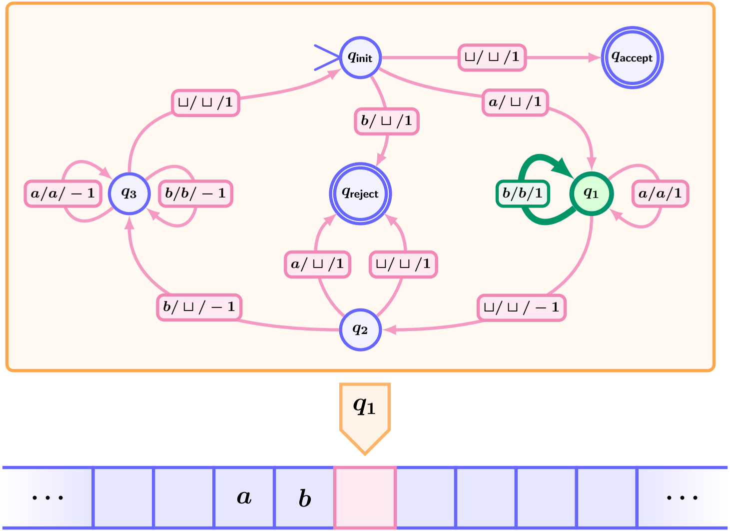

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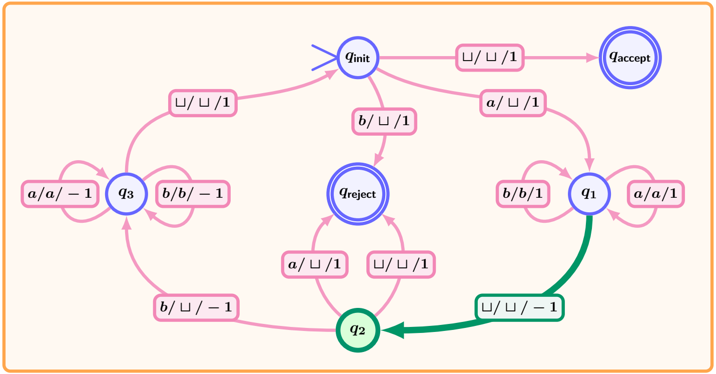

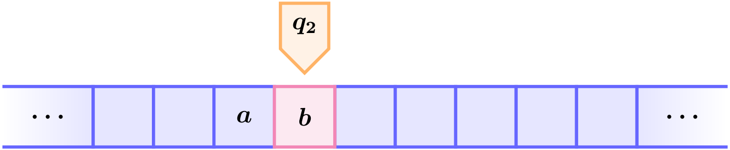

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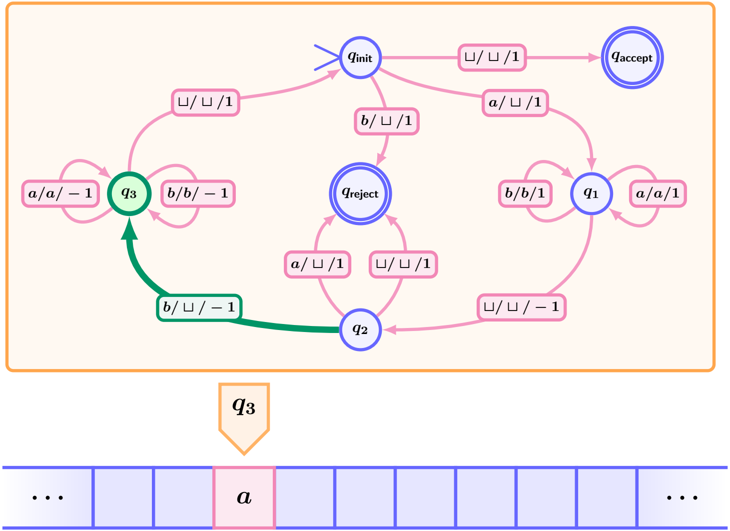

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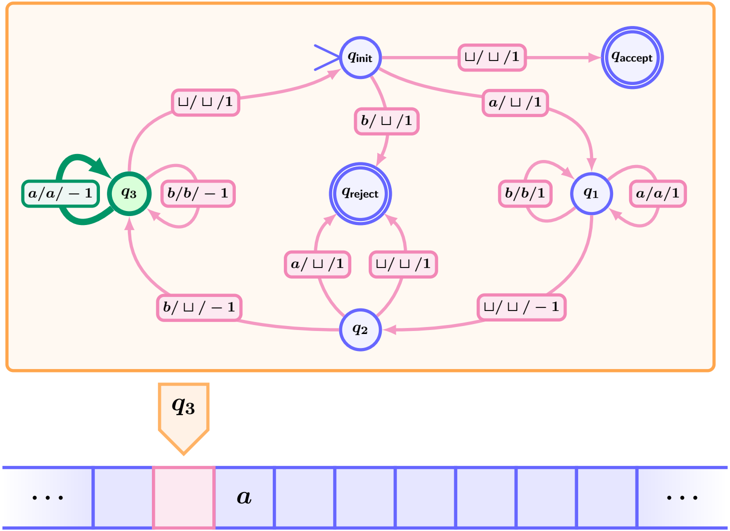

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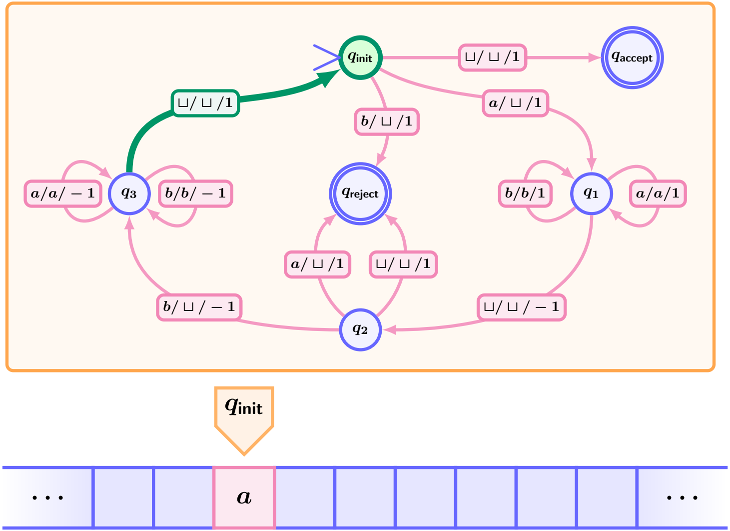

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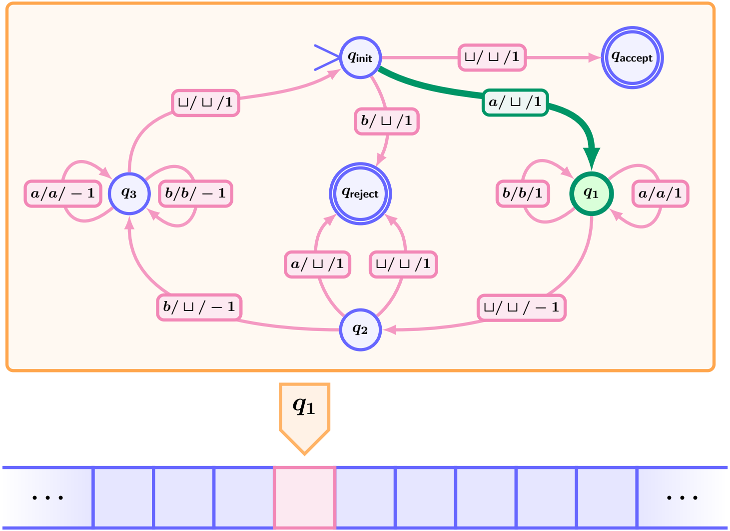

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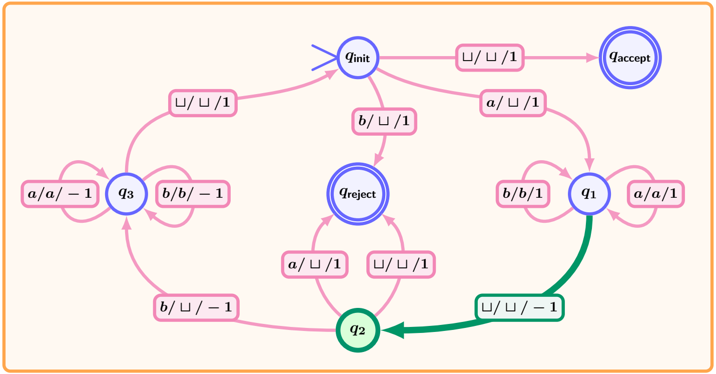

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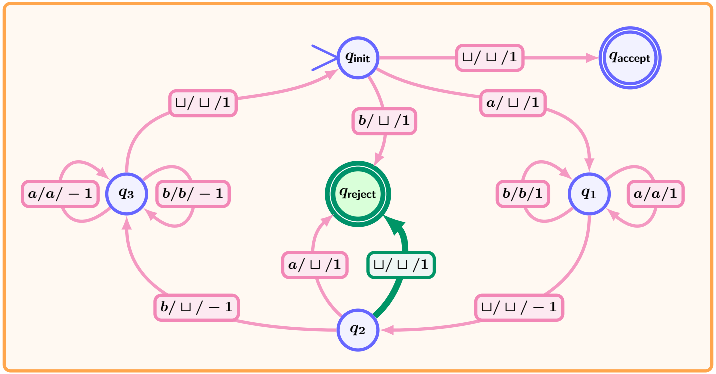

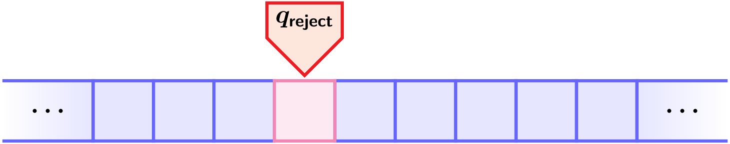

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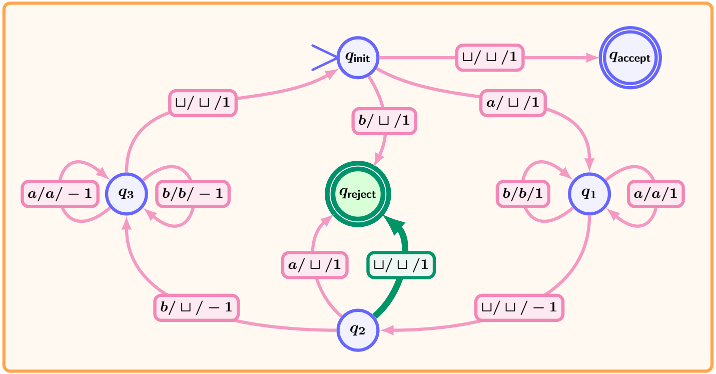
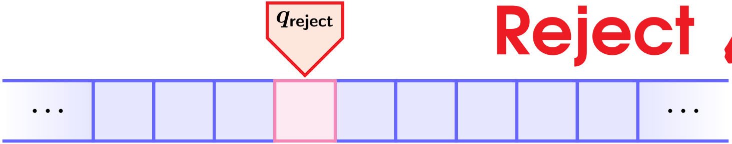
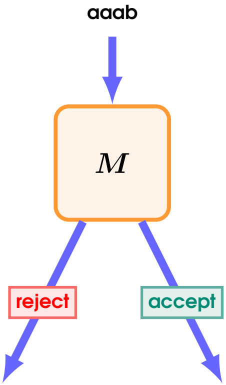


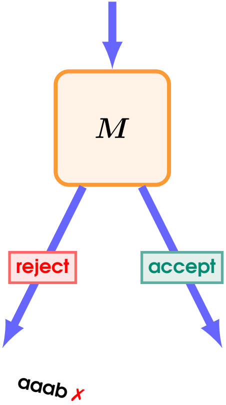


Reject ✗

- **Decision Procedure**

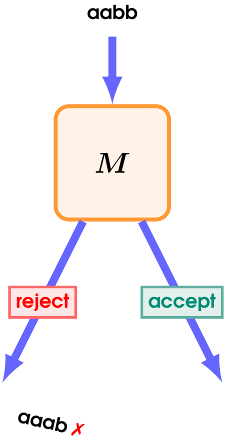


- **Decision Procedure**



Turing Machines

- **Decision Procedure**



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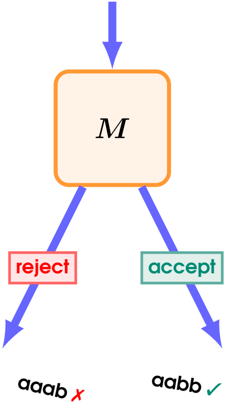
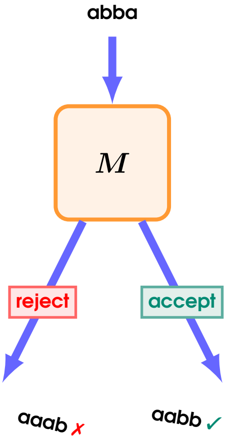
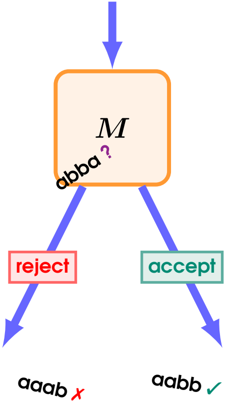


- **Decision Procedure**



- **Decision Procedure**



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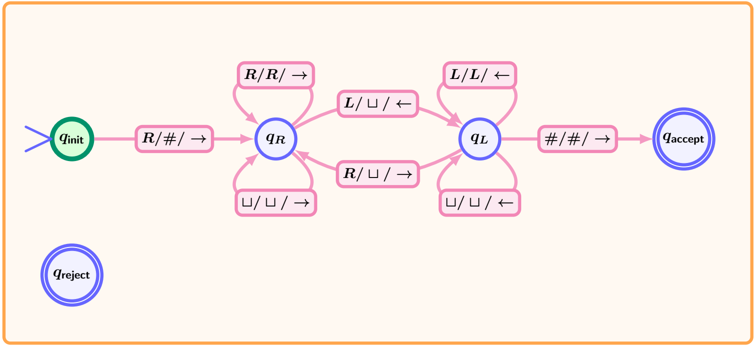

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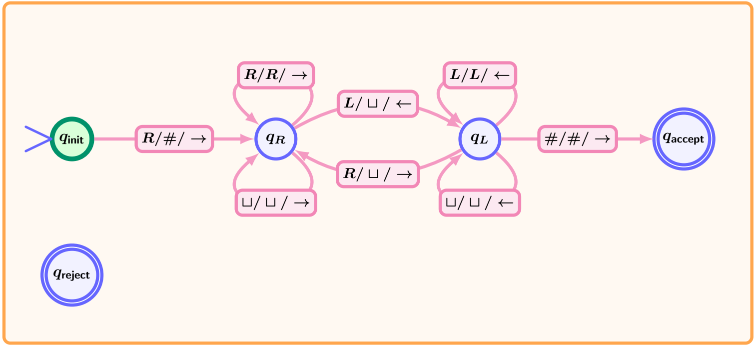

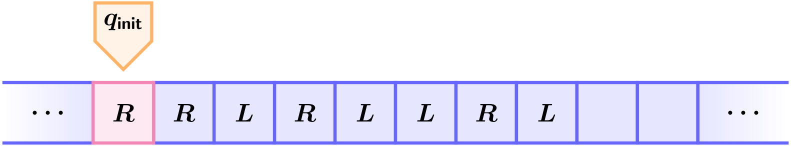

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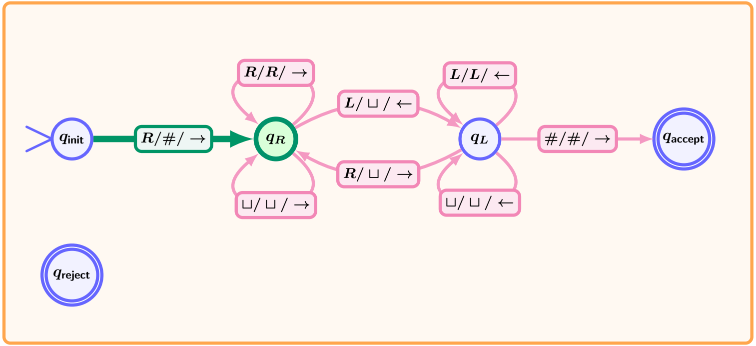

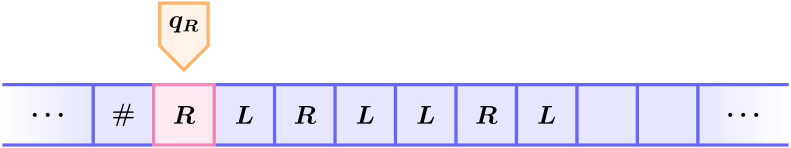

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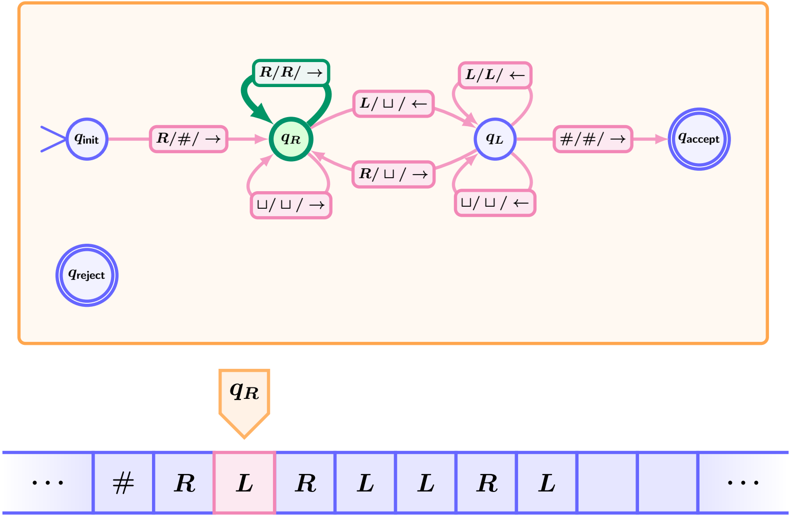

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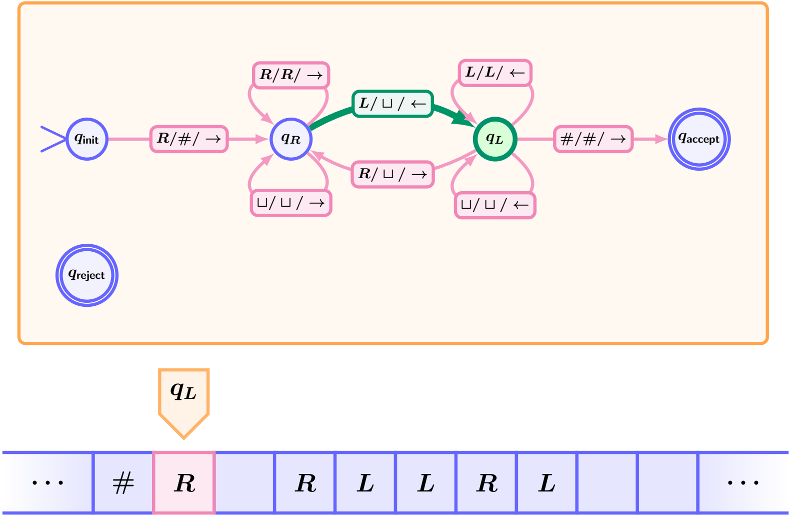

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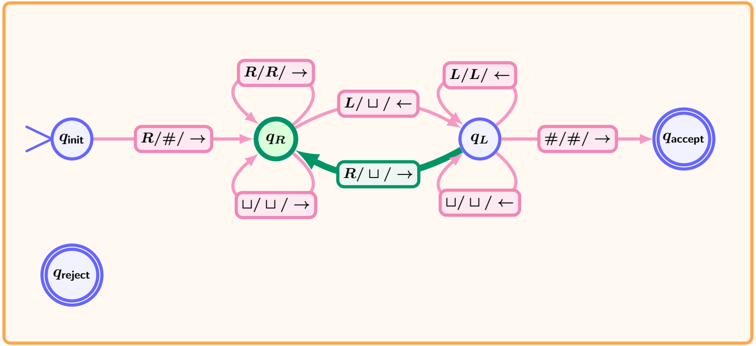

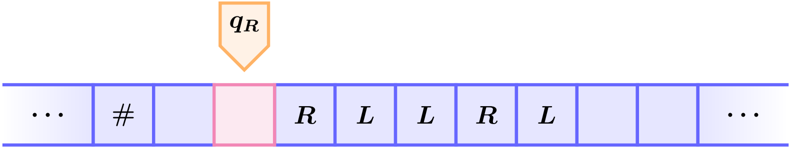

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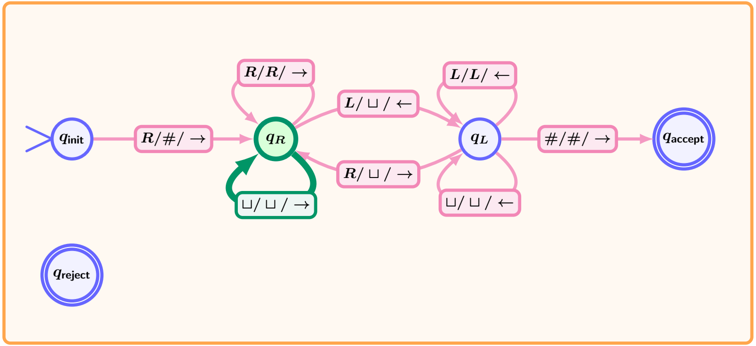

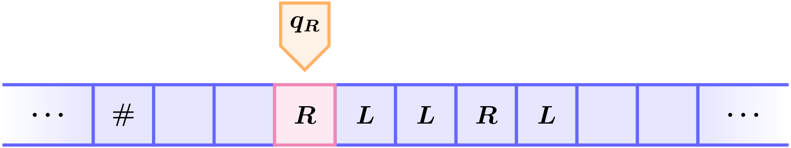

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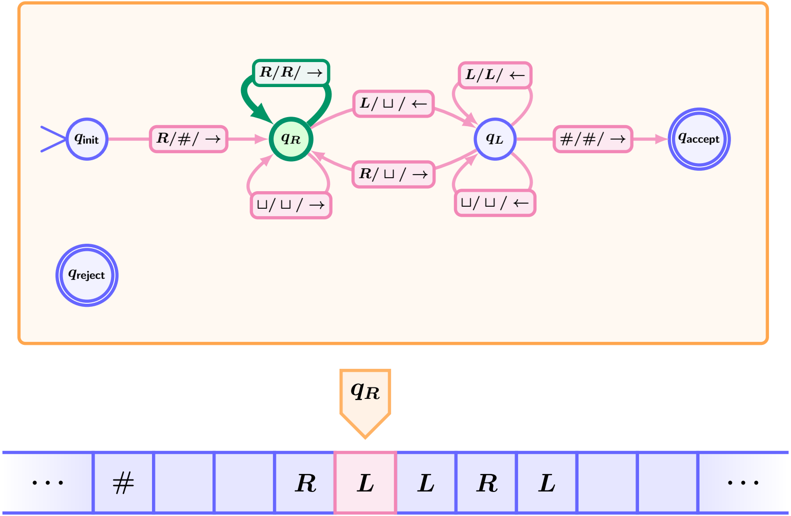

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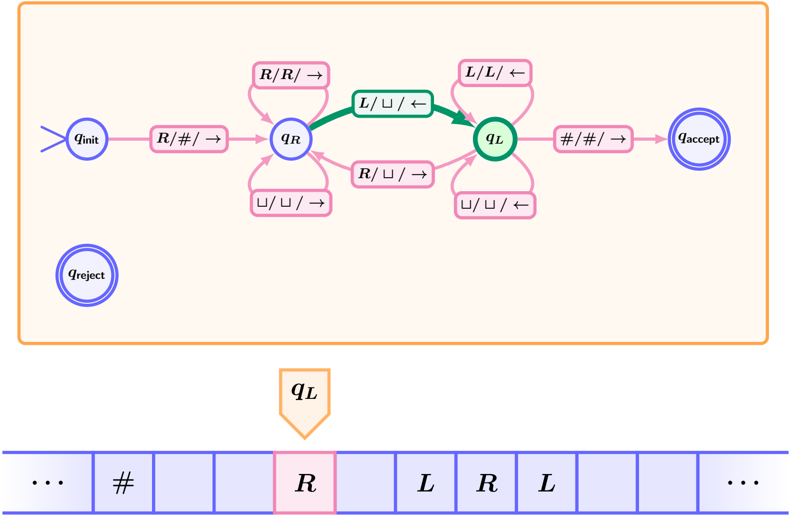

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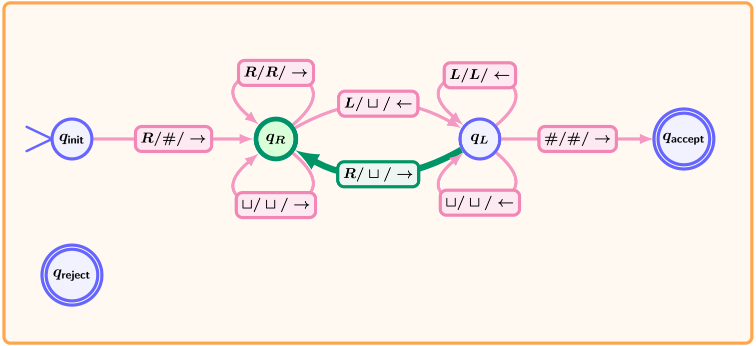

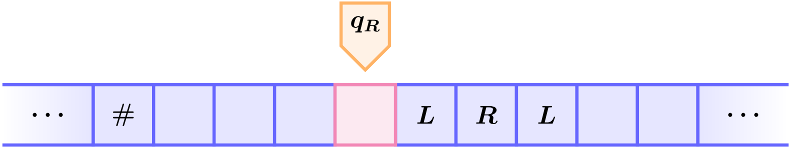

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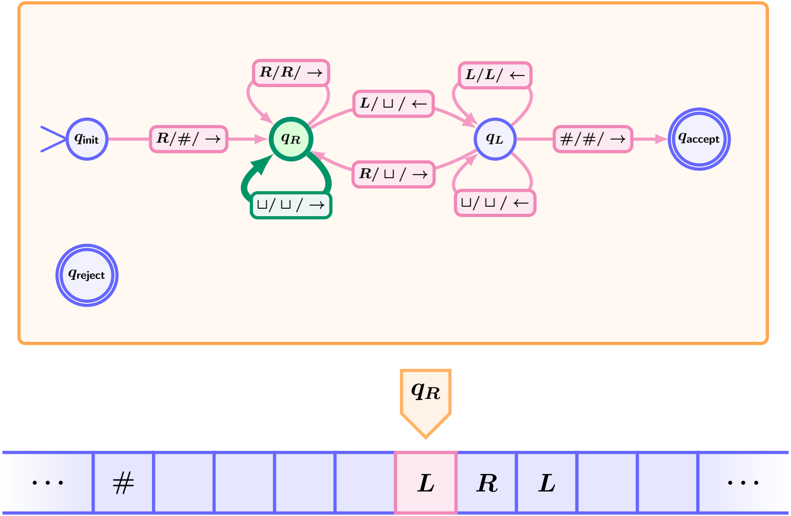

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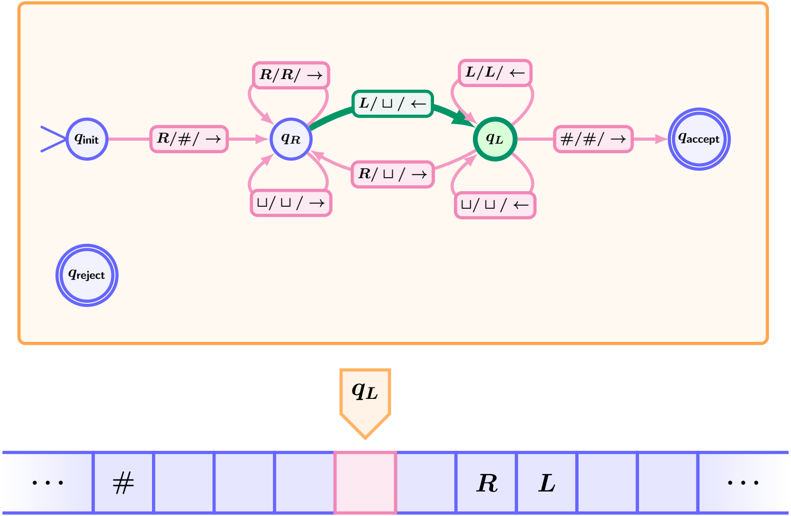

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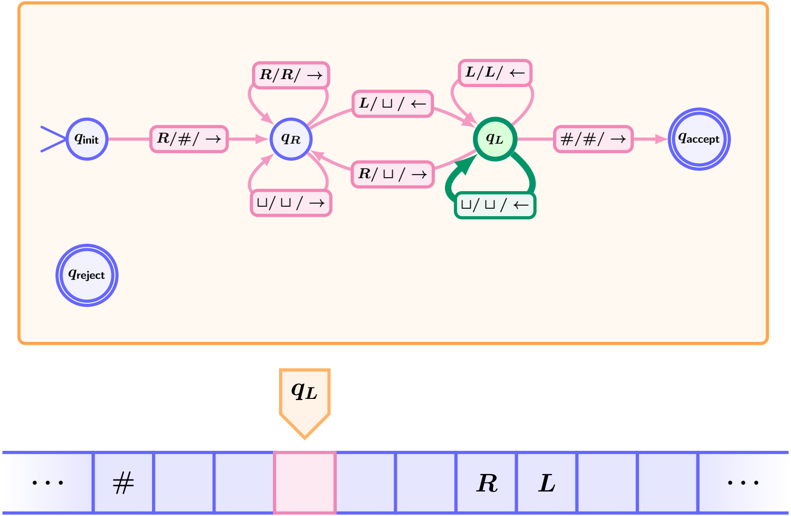

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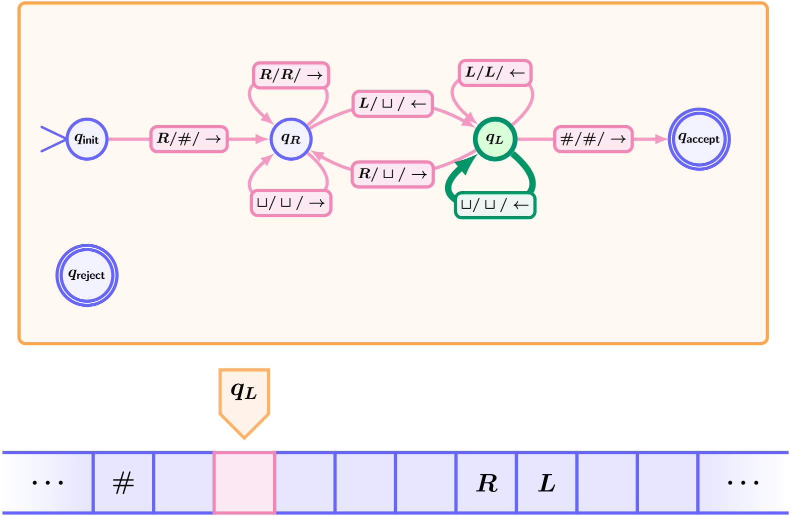

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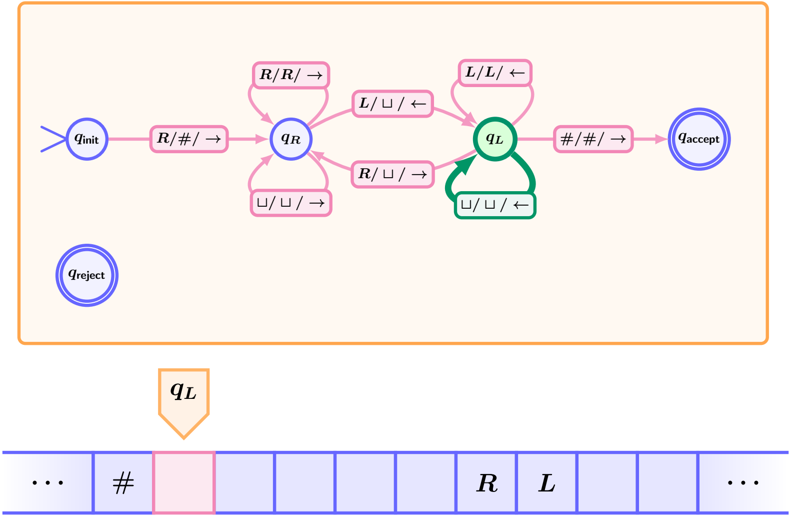

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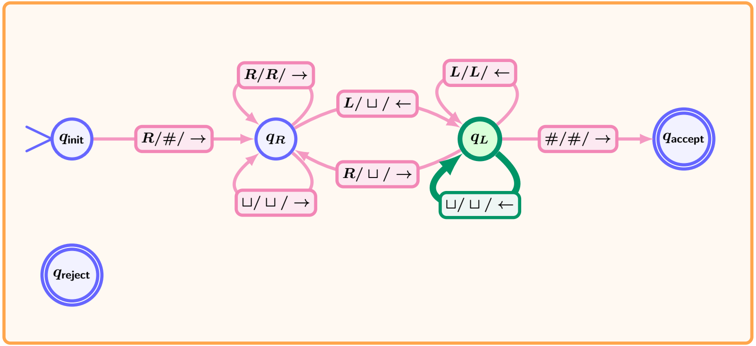

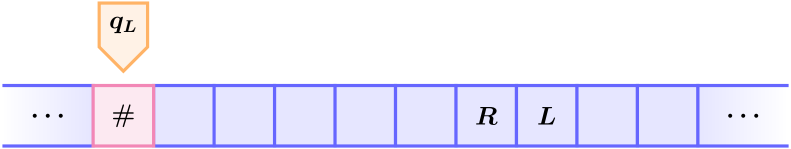

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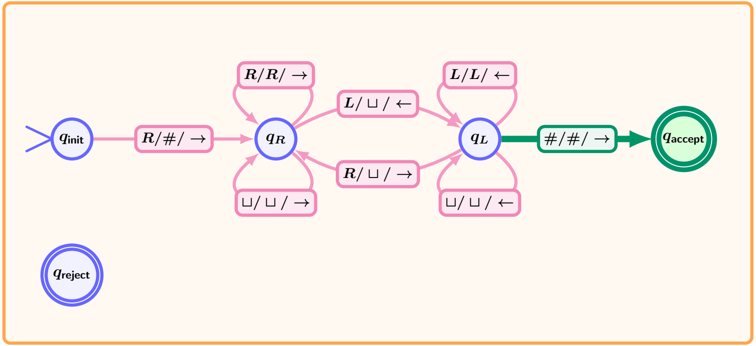

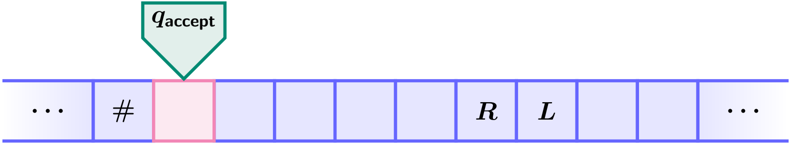

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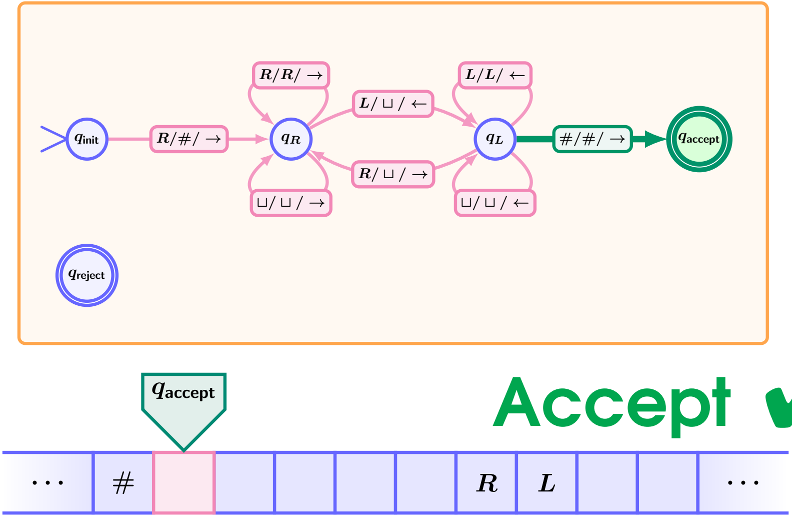
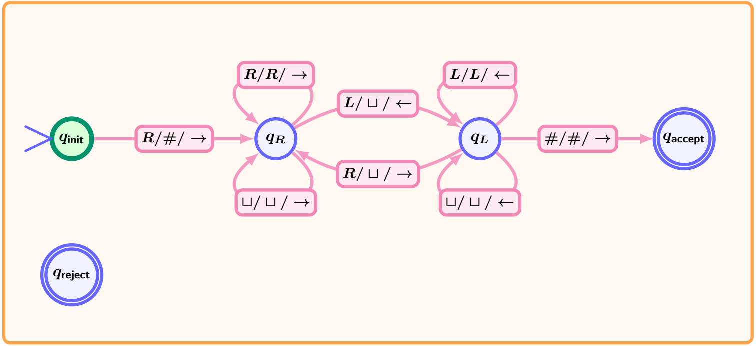
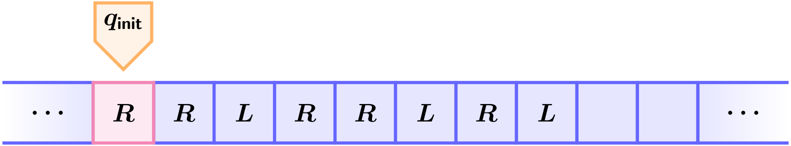


Accept ✓

Turing Machines





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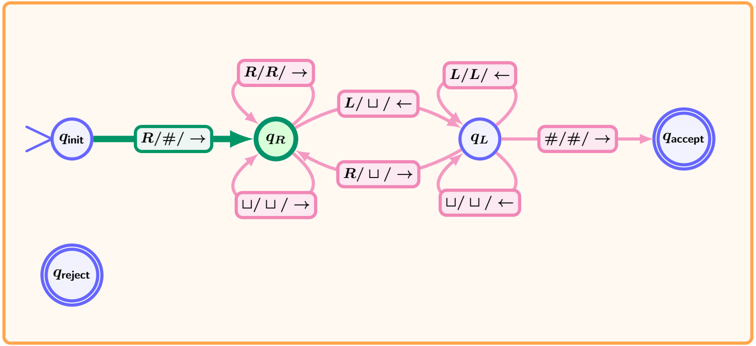
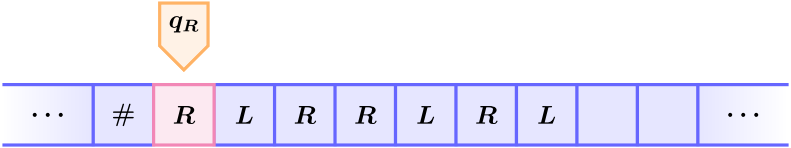
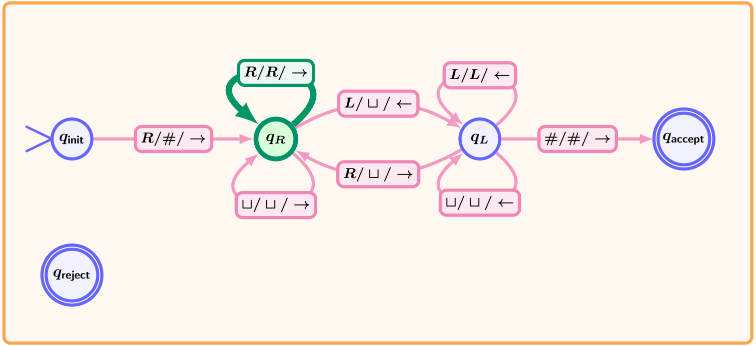
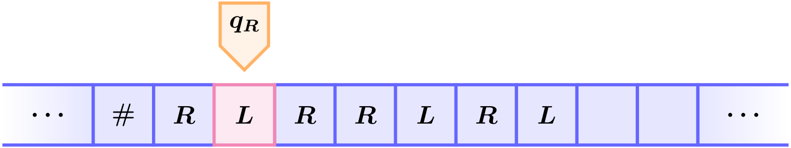




Turing Machines





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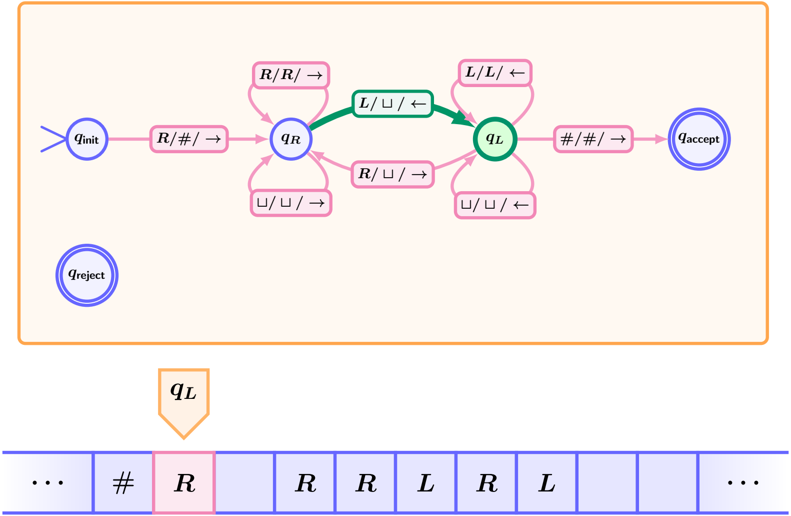

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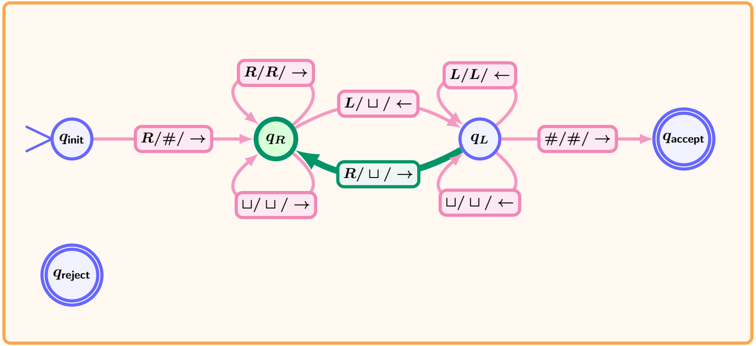

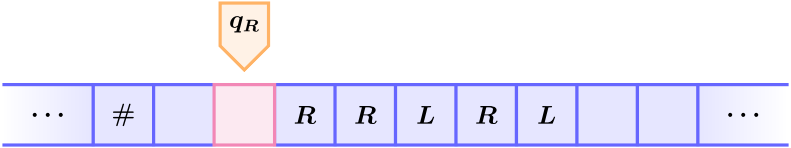

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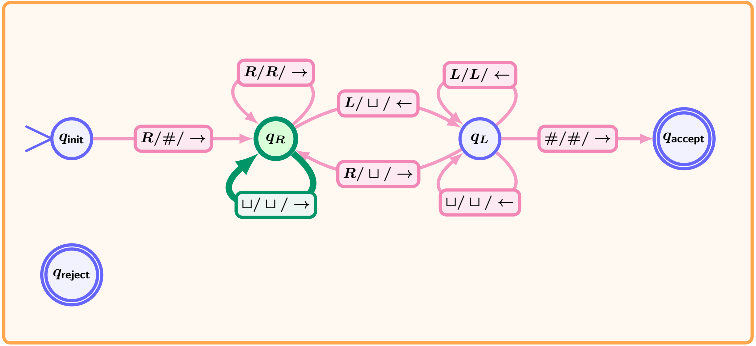

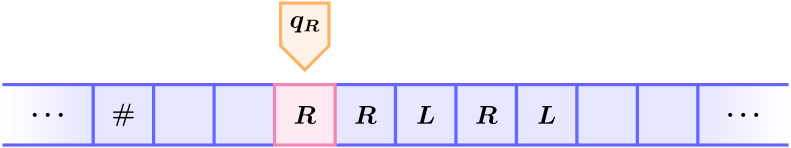

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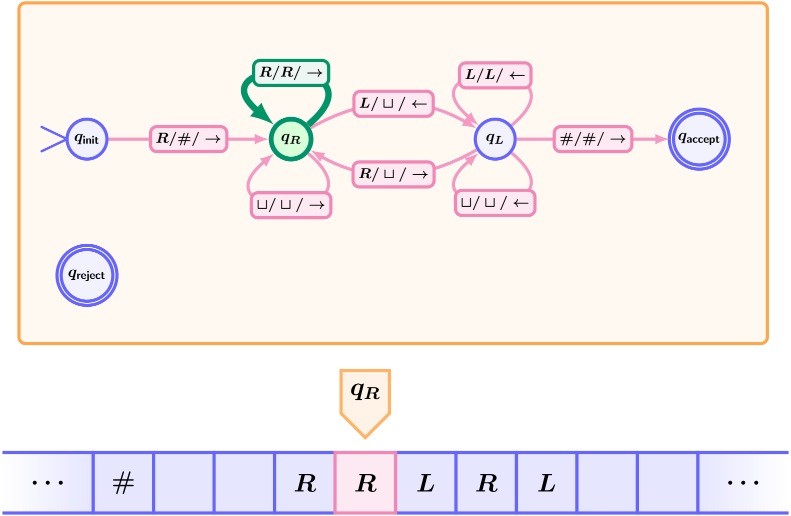

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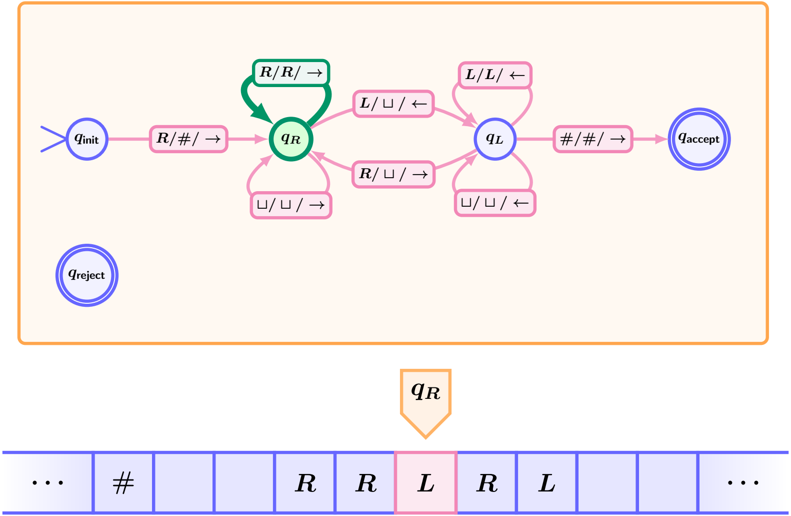

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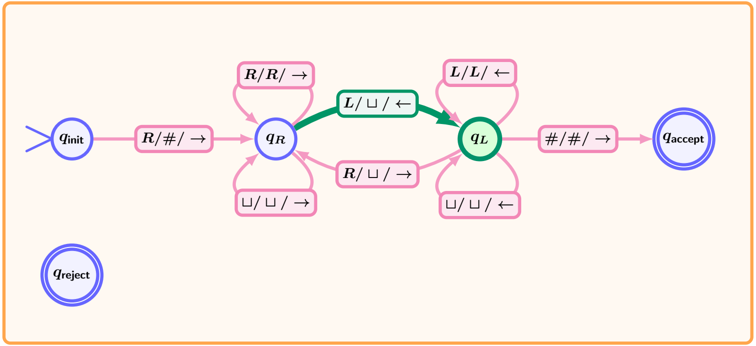

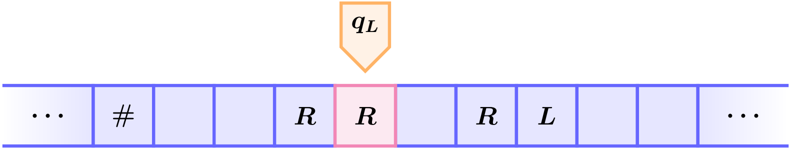

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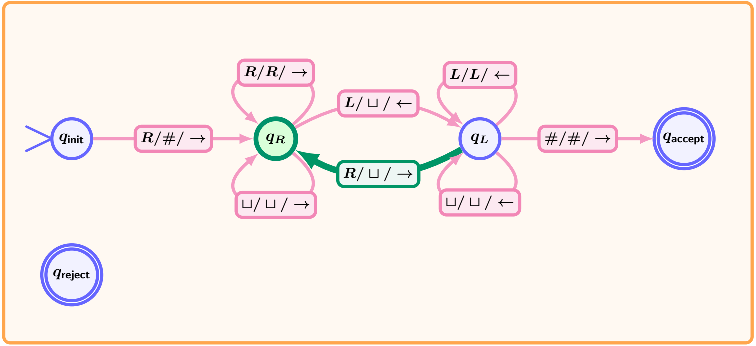

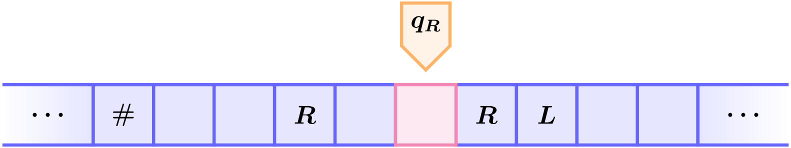

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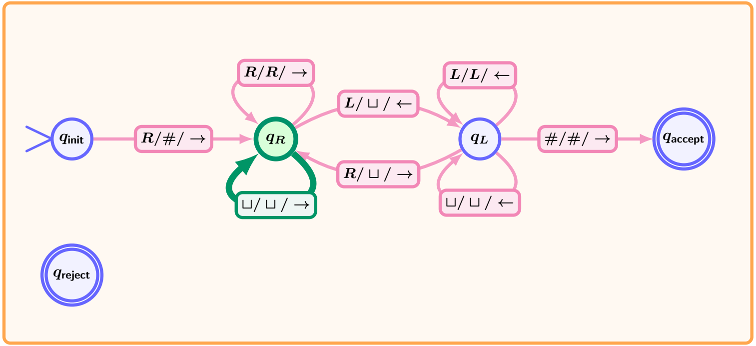

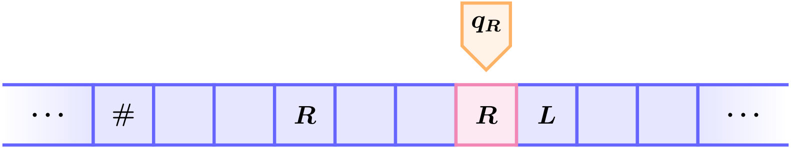

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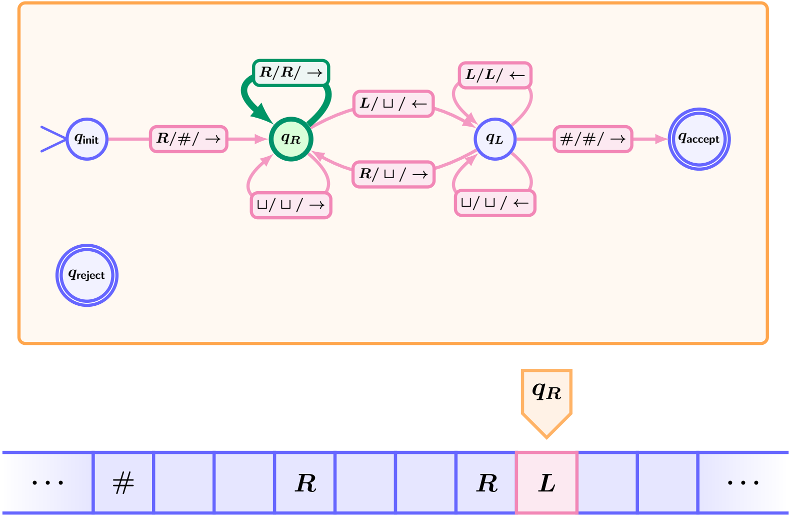

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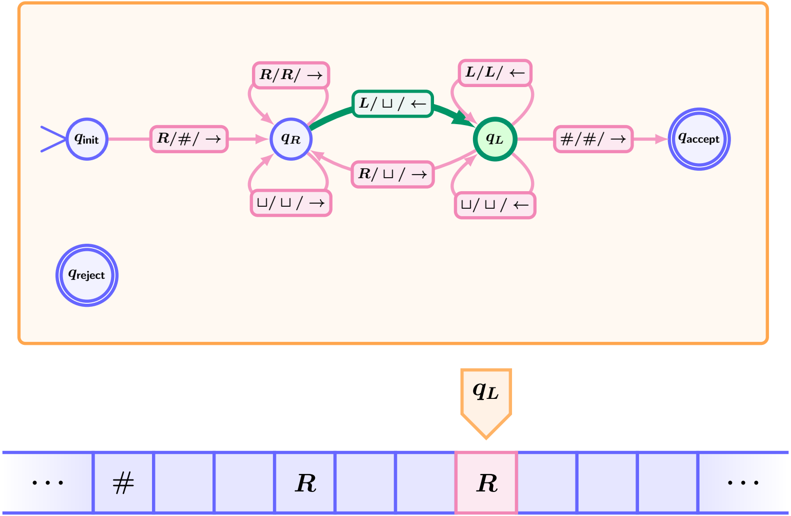

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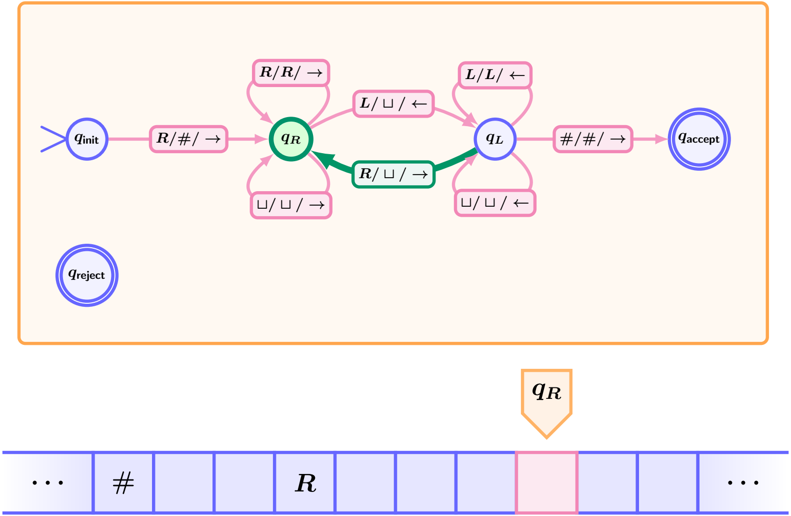

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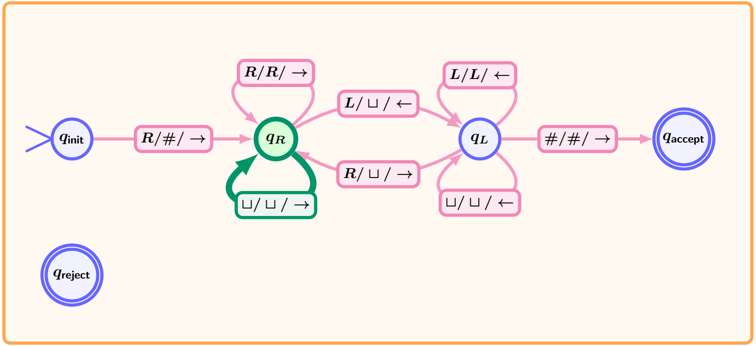

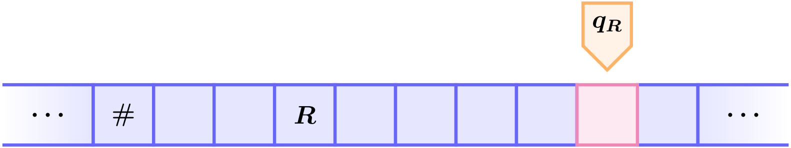

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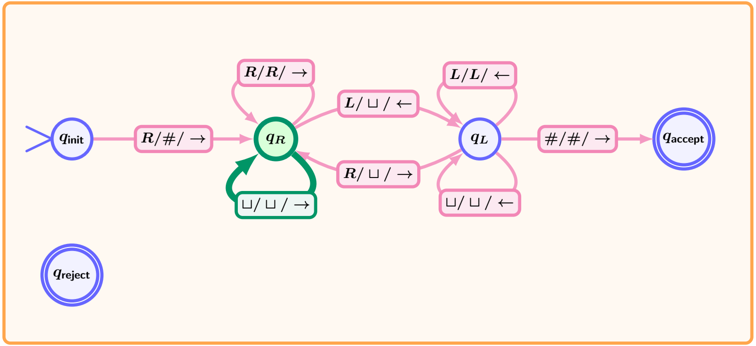

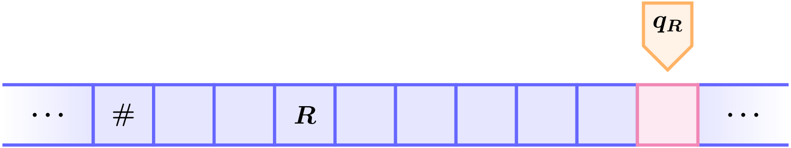

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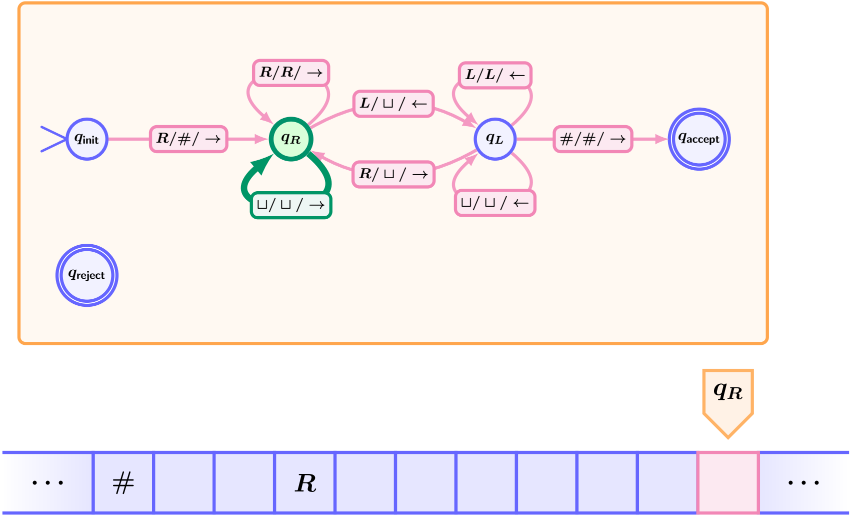

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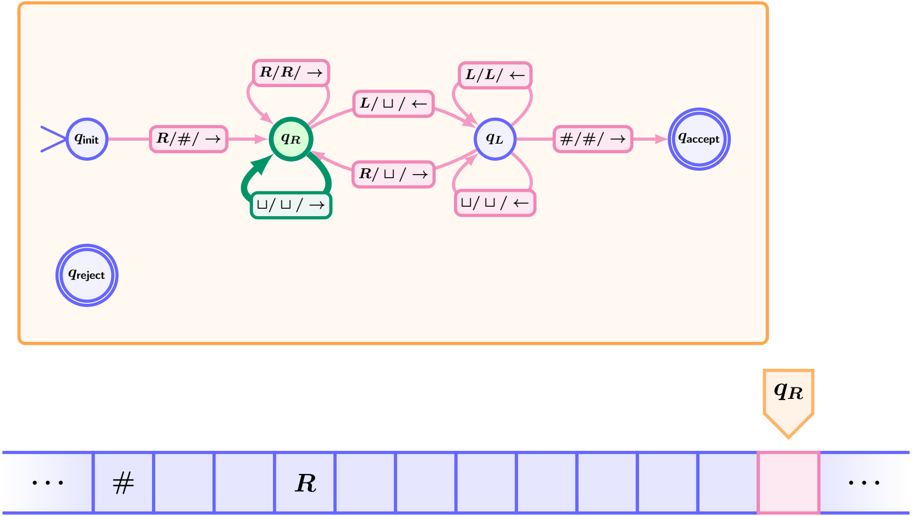

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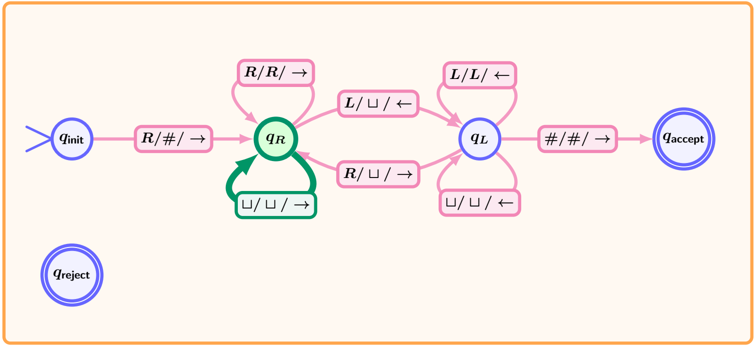

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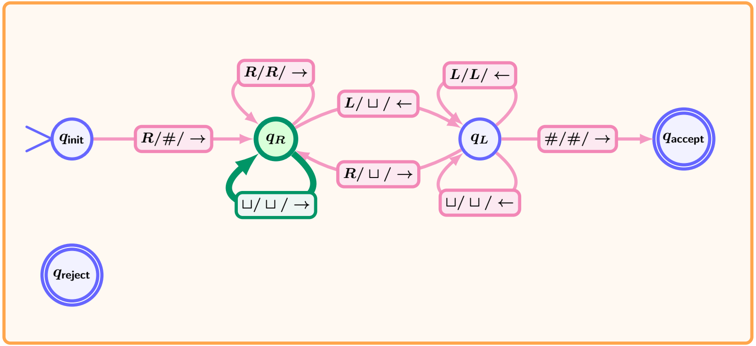

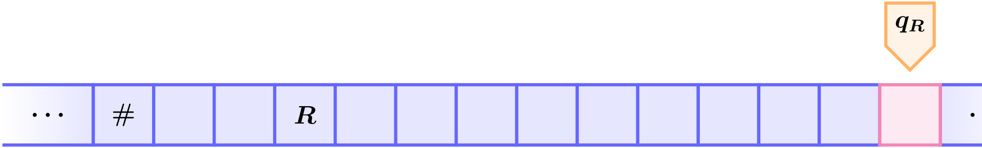

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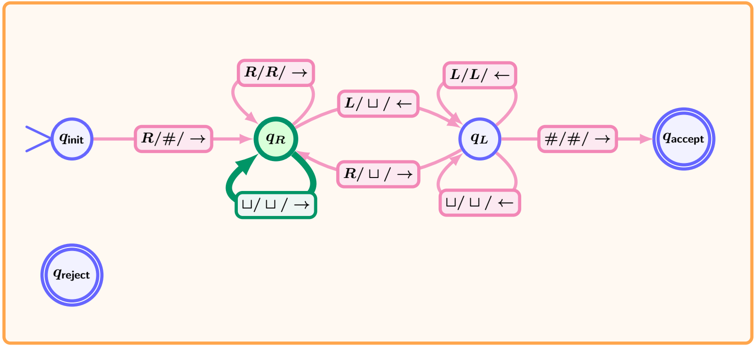

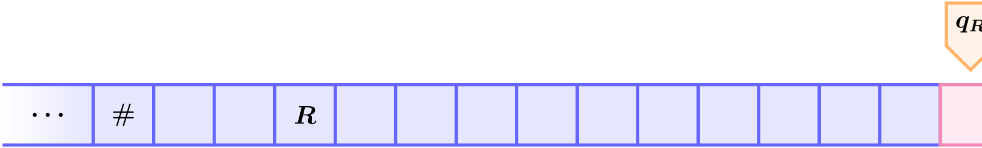

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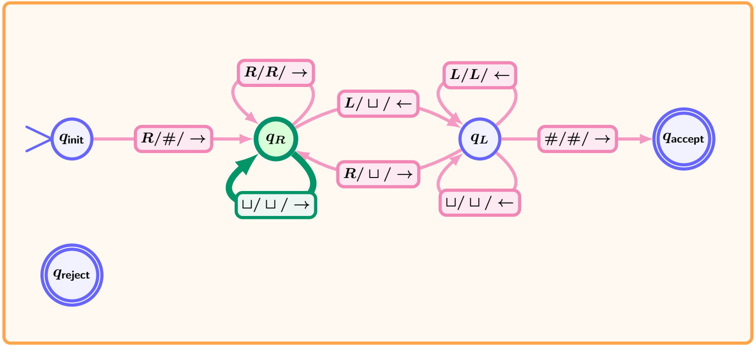

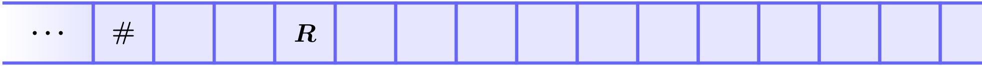

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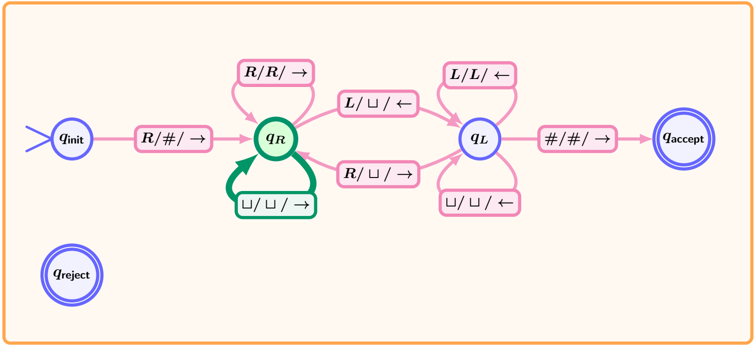
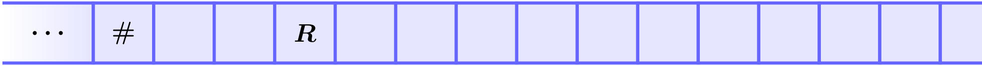
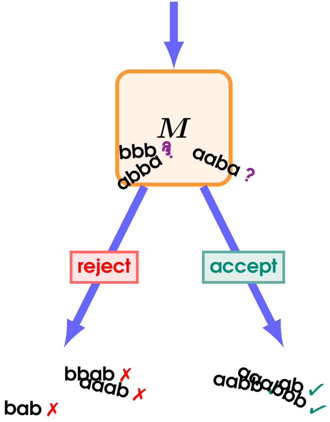


Undecided ?



- **Decision Procedure**



- **Decision Procedure**
  - We use the following notation as a **shorthand** to describe the behaviour of M on input word w

$$M(w) := \begin{cases} 1 & \text{if } M \text{ accepts } w \\ 0 & \text{if } M \text{ rejects } w \\ \uparrow & \text{if } M \text{ does not halt on } w \end{cases}$$

- **Decision Procedure**
  - A machine M is said to **decide** a language L if it is:
    - **Sound (w.r.t L)** — $\boxed{\text{If } M(w) = 1 \text{ then } w \in L}$
    - **Complete (w.r.t L)** — $\boxed{\text{If } w \in L \text{ then } M(w) = 1}$
    - **Terminating** — $\boxed{T_M(w) < \infty \text{ is finite for all } w \in \Sigma^*}$

- **Decision Procedure**
  - The languaged decided by M is the set

$$\textbf{Language}\,(M) \;=\; \{w \in \Sigma^* \;:\; M(w)\text{ accepts } w\}$$

End of Slides!