5CCS2FC2: Foundations of Computing II

Divide-and-Conquer and Recursion

Week 3

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Divide-and-Conquer

Divide-and-Conquer Techninque

Divide-and-Conquer

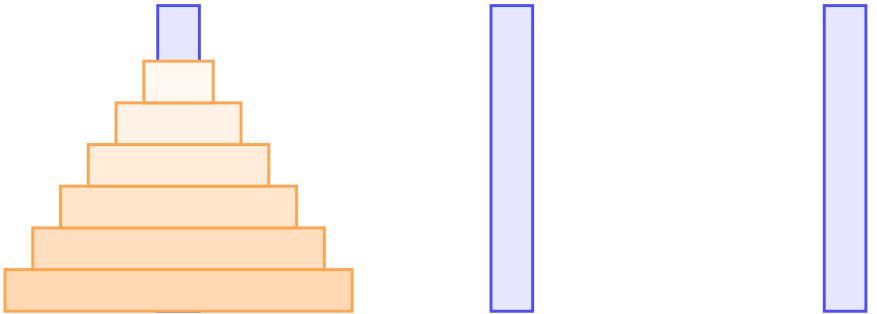
Divide) Divide the problem into several 'self-similar' but smaller sub-problems,

Conquer) Solve these sub-problems recursively,

Combine) Recombine the sub-problems into a solution for the whole problem.

The Towers of Hanoi

The Towers of Hanoi



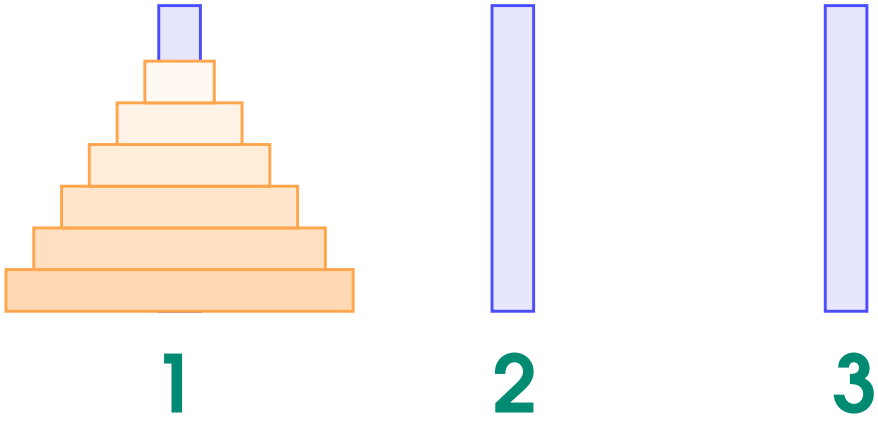
The Towers of Hanoi

Constraint 1) You can only move **one block** at a time.

Constraint 2) You cannot place any block on top of a **smaller** block.

The Towers of Hanoi

MoveTower(n, 1, 3):



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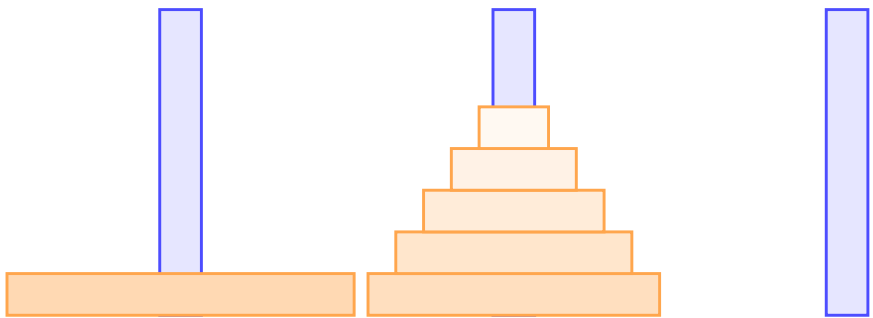

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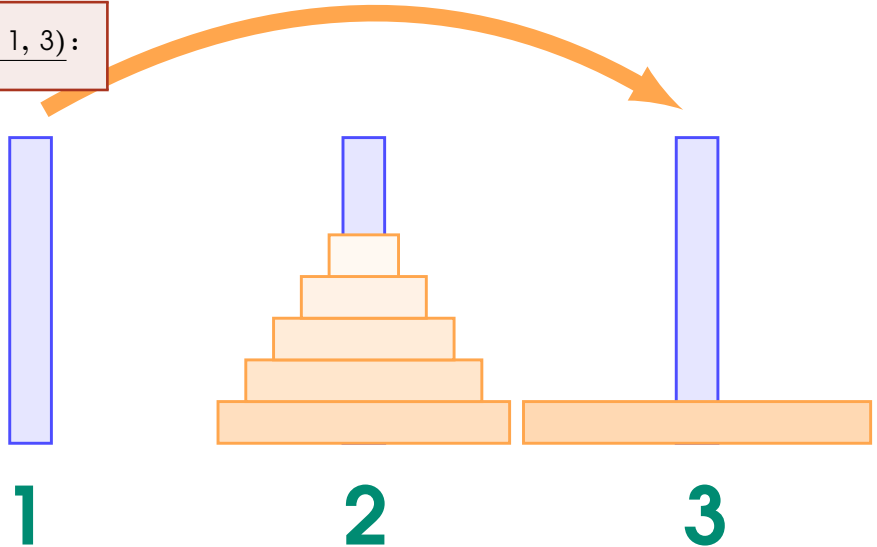

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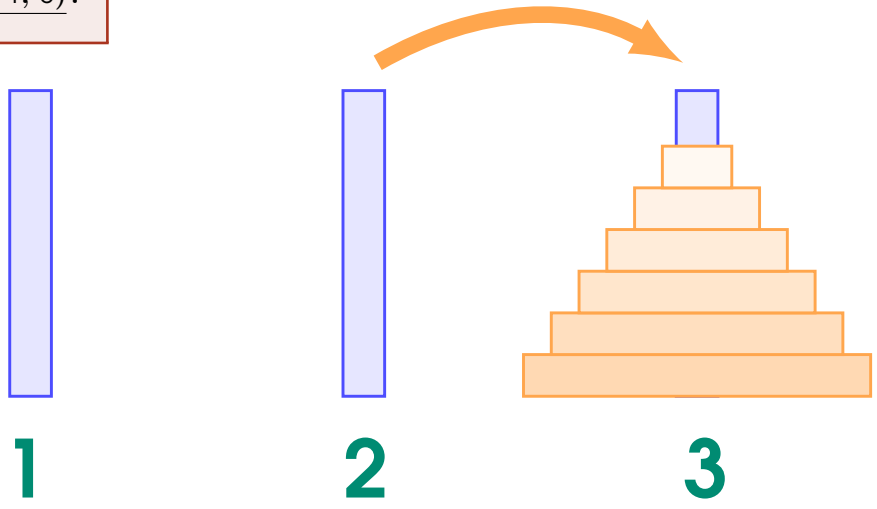

The Towers of Hanoi

MoveTower(n, 1, 3):

1. **If** $n = 1$ **then** Move(1, 3)
2. **Else**
    3. MoveTower($n - 1$, 1, 2)
    4. Move(1, 3)
    5. MoveTower($n - 1$, 2, 3)

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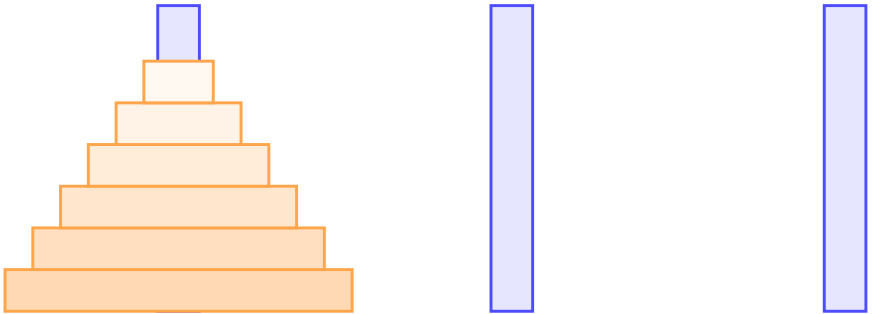

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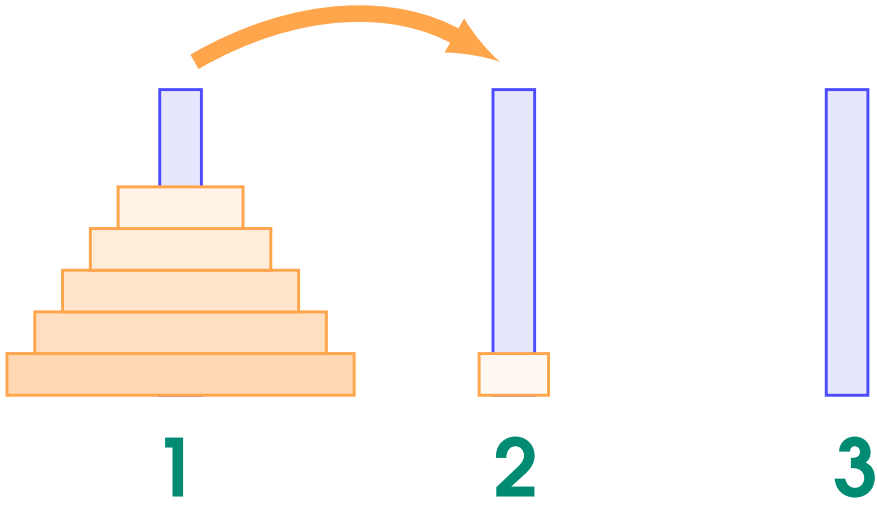

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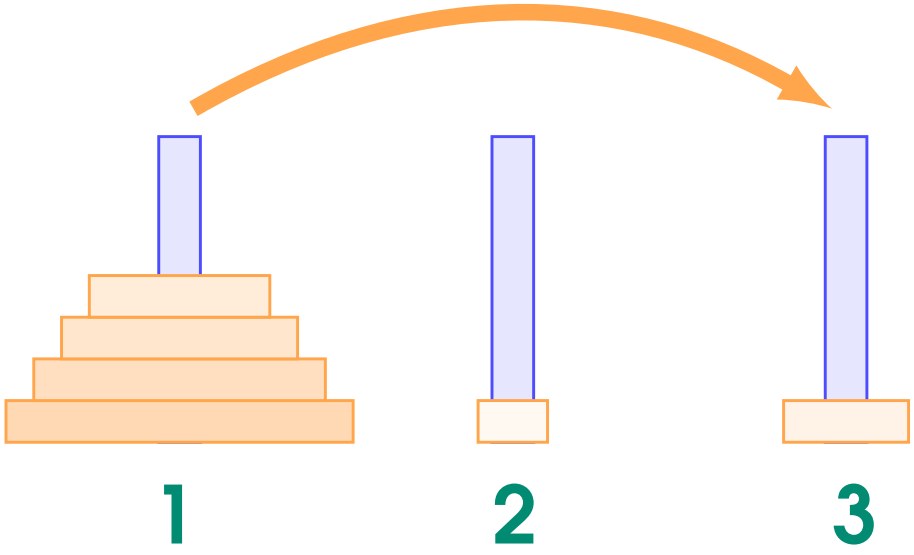

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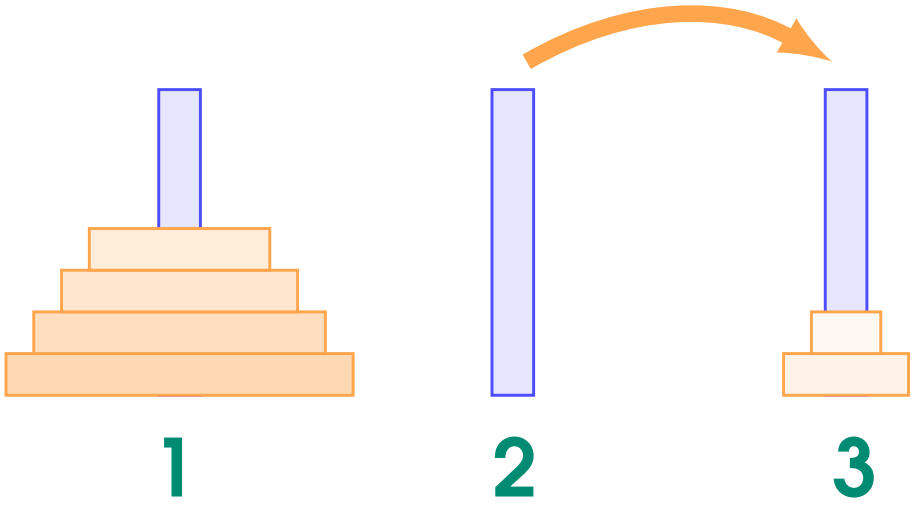

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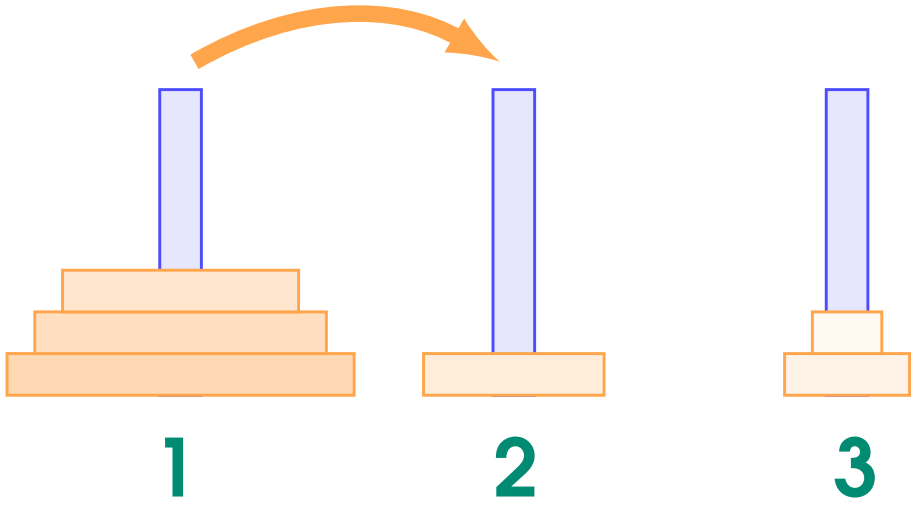

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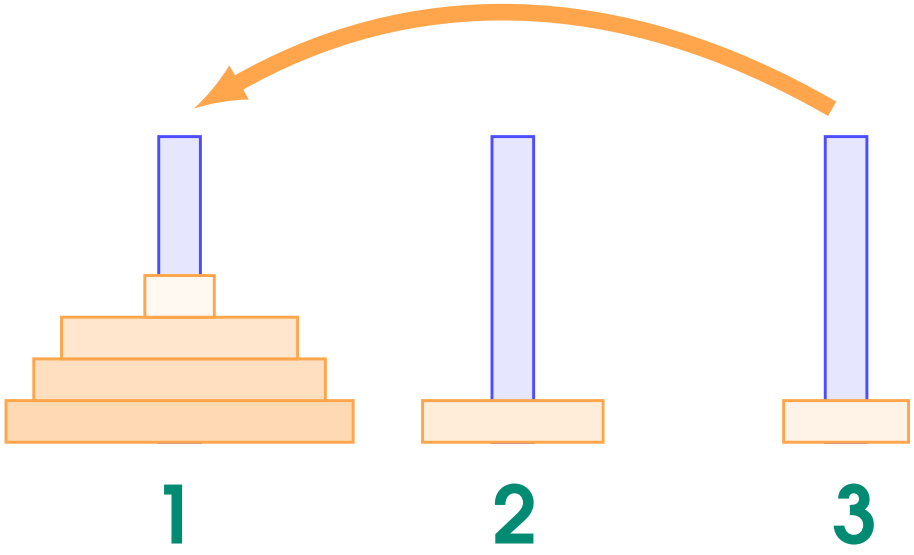

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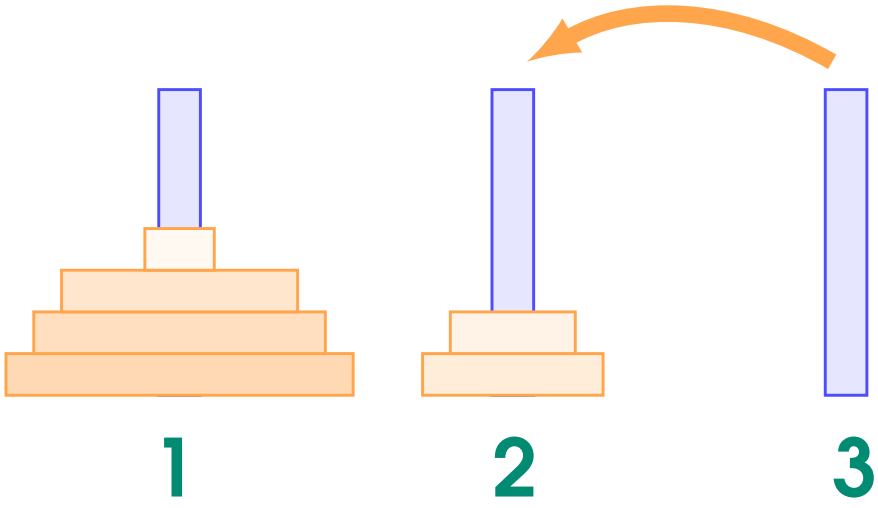

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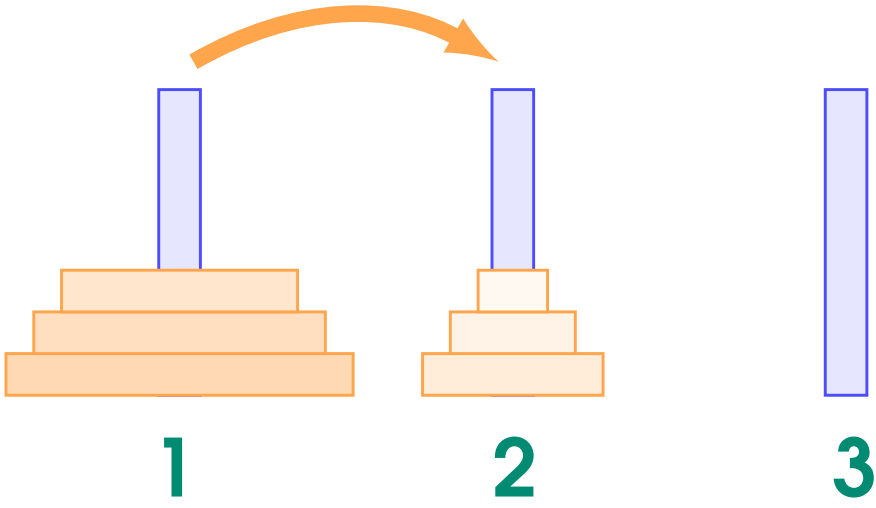

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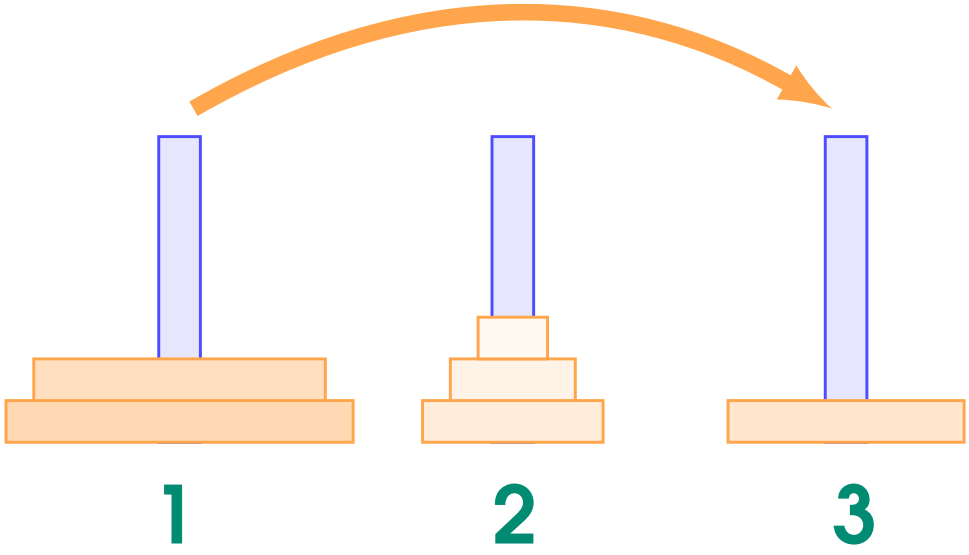

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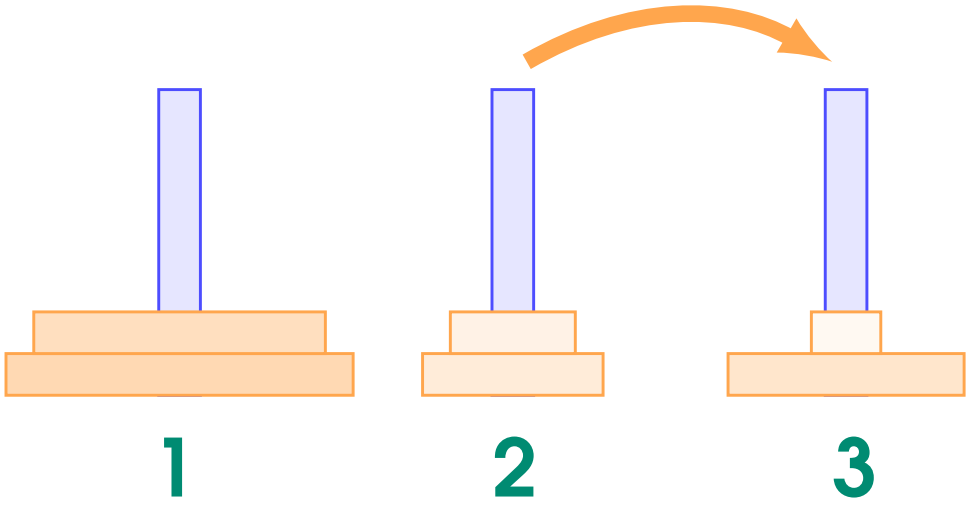

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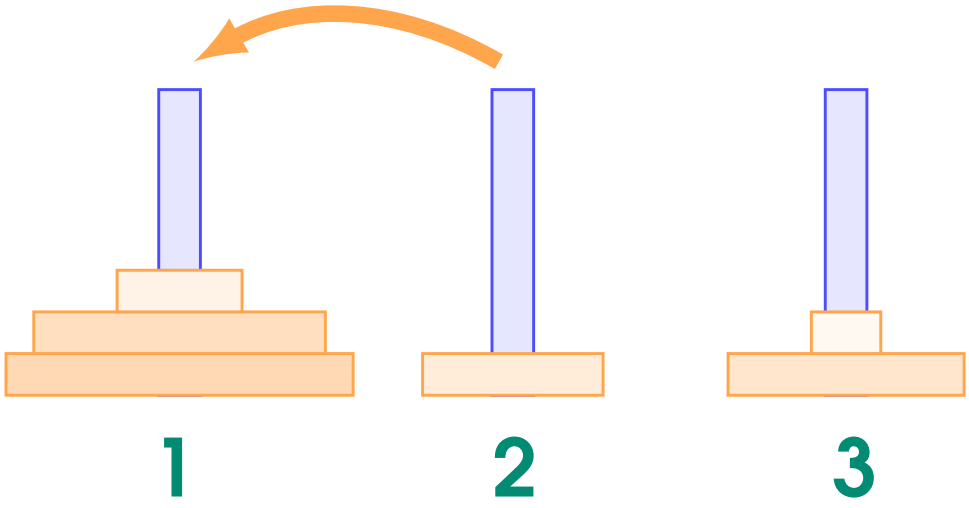

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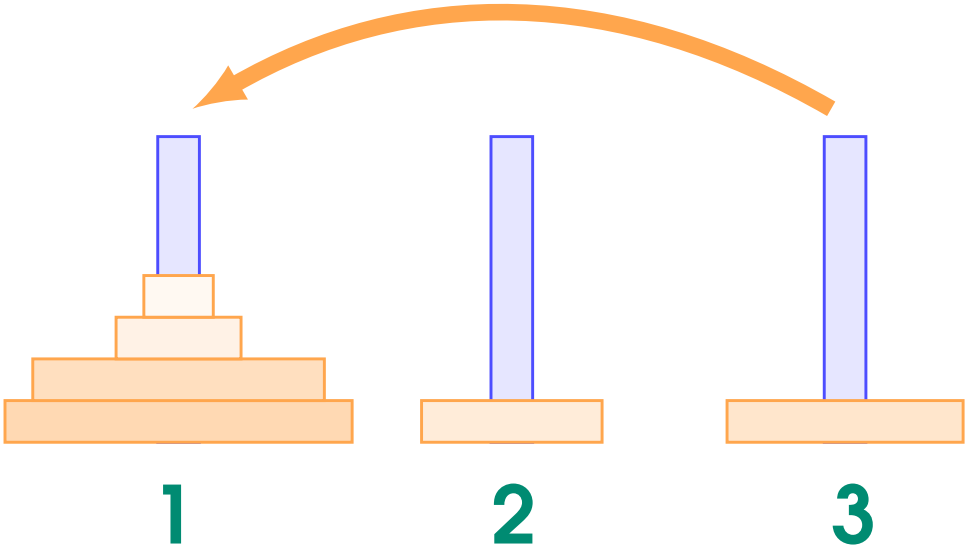

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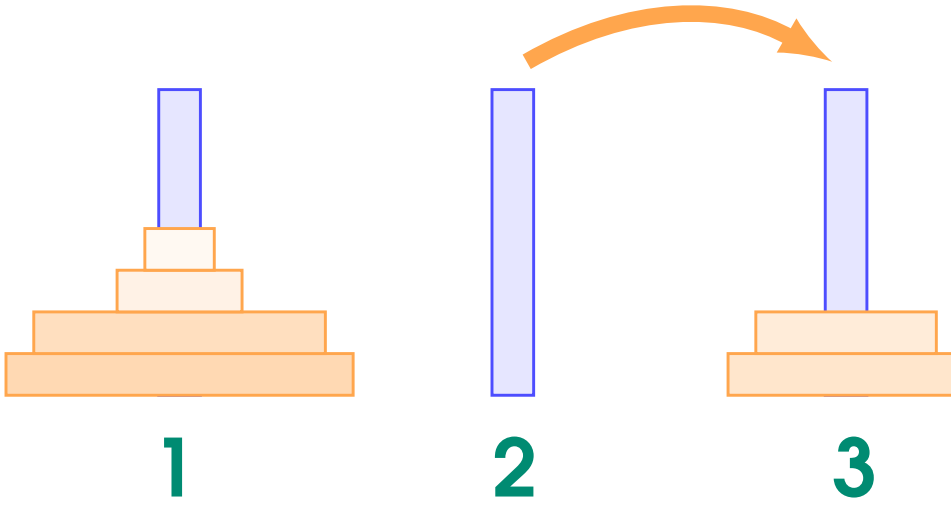

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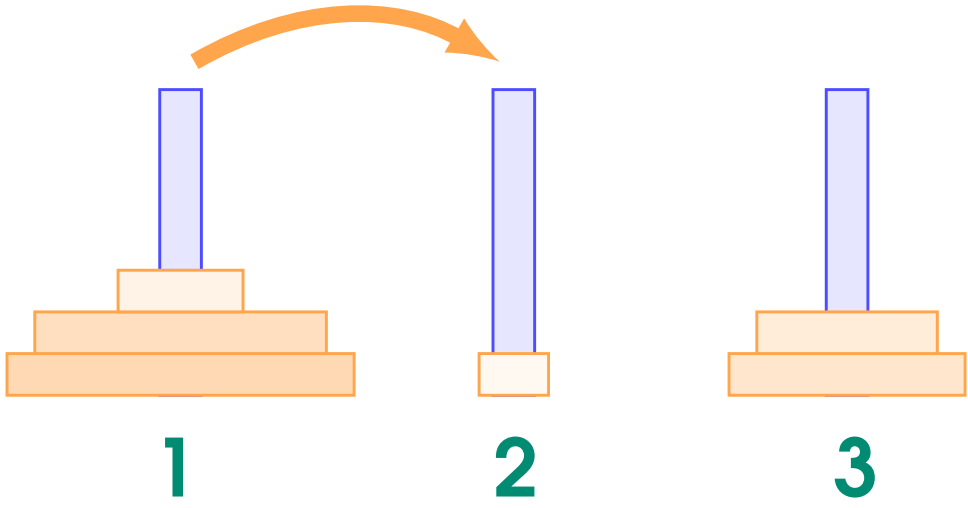

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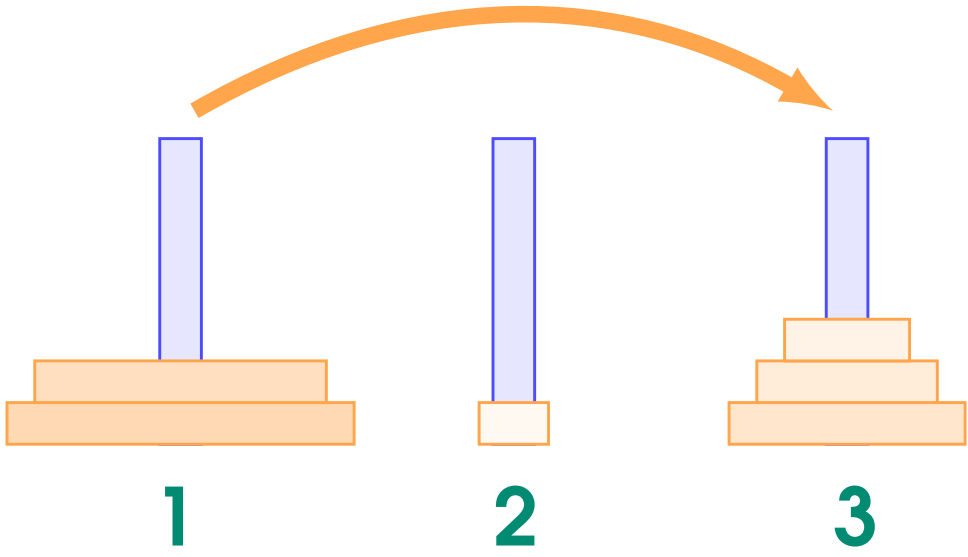

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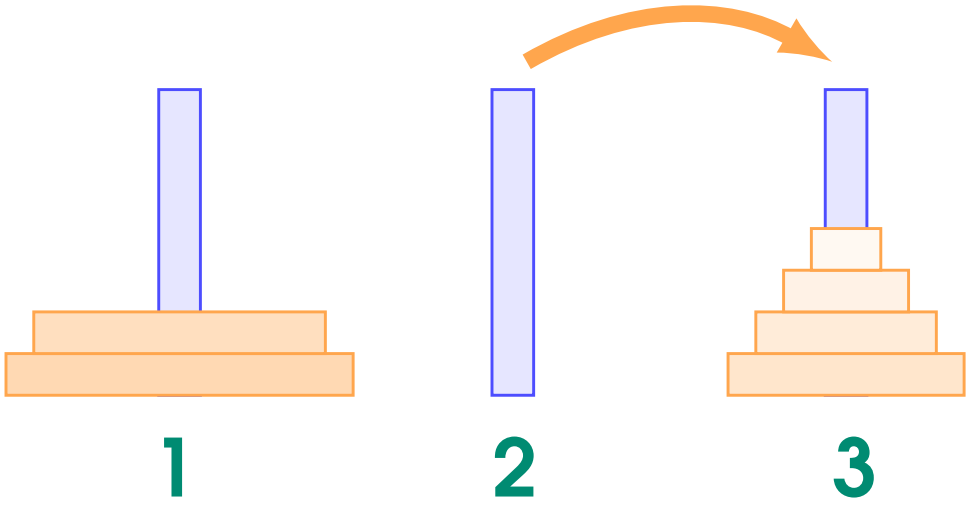

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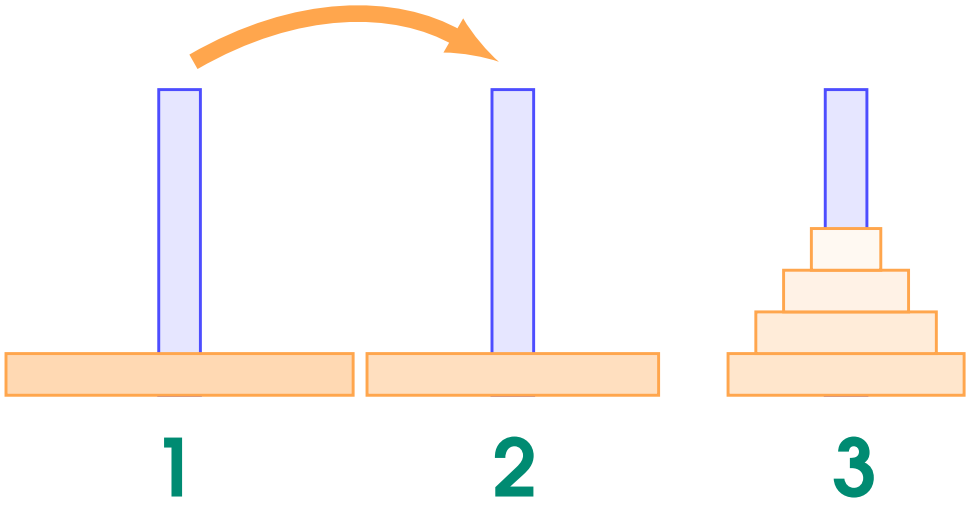

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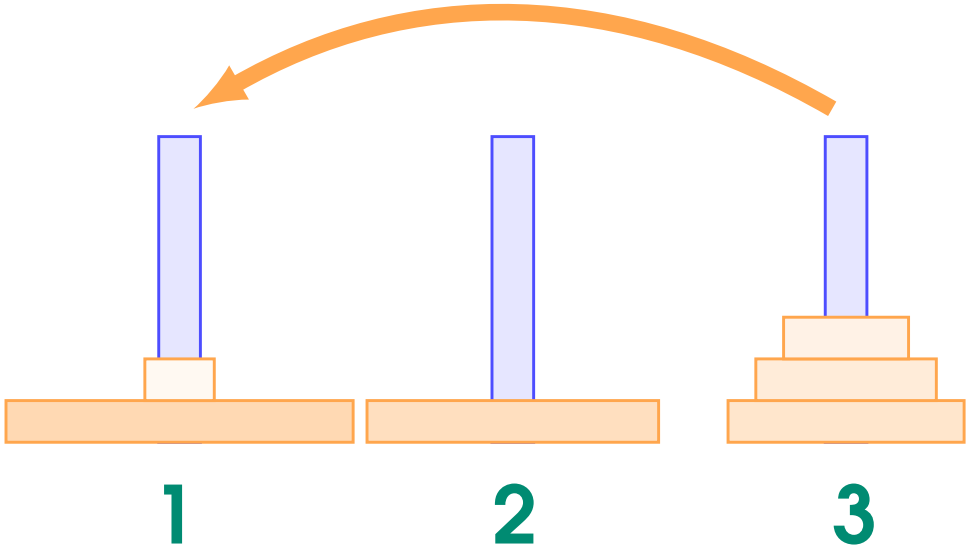

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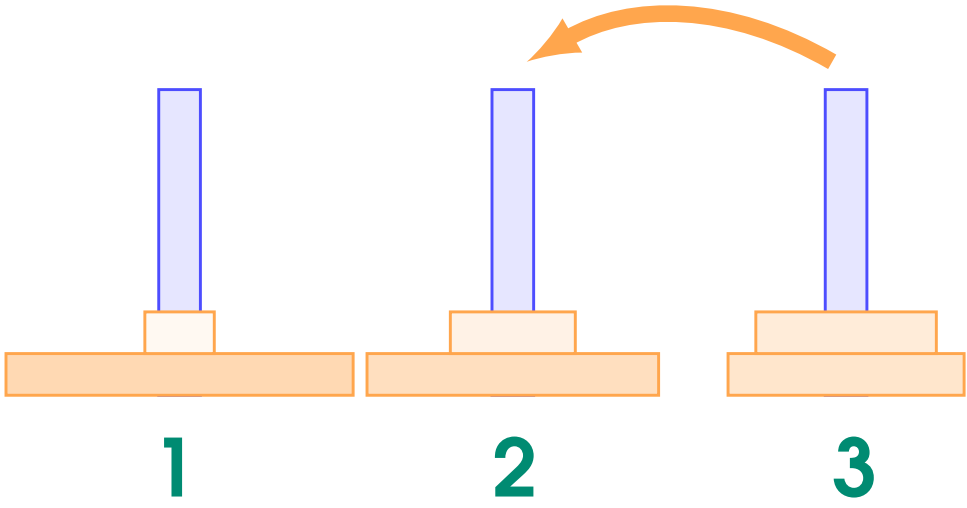

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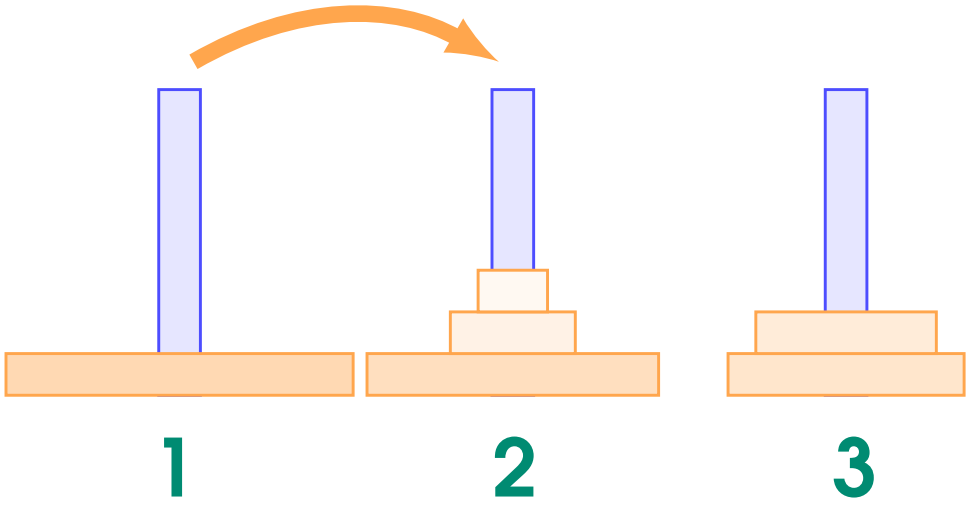

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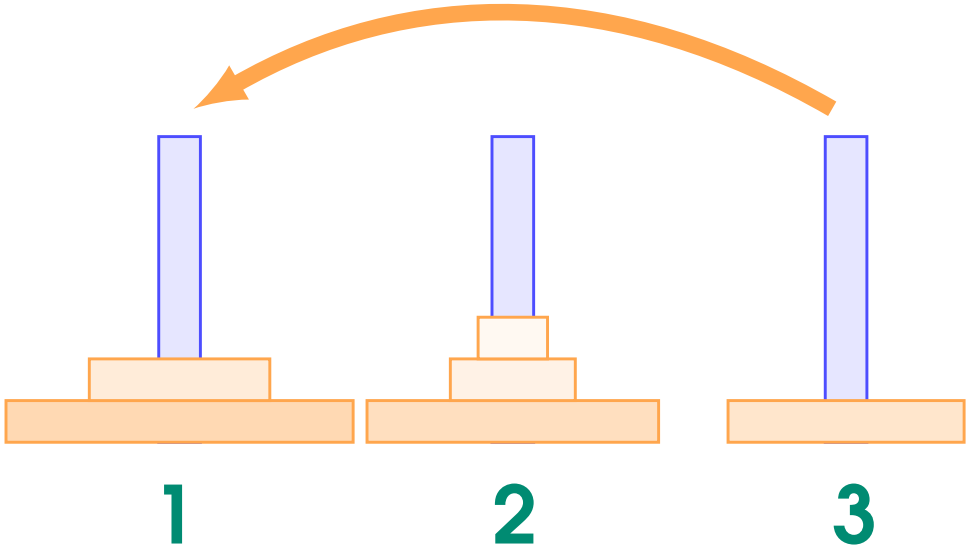

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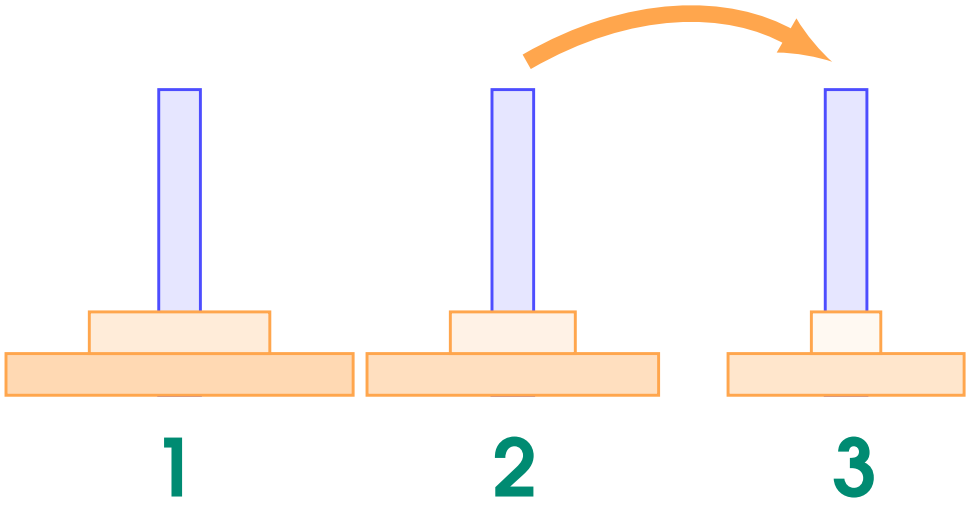

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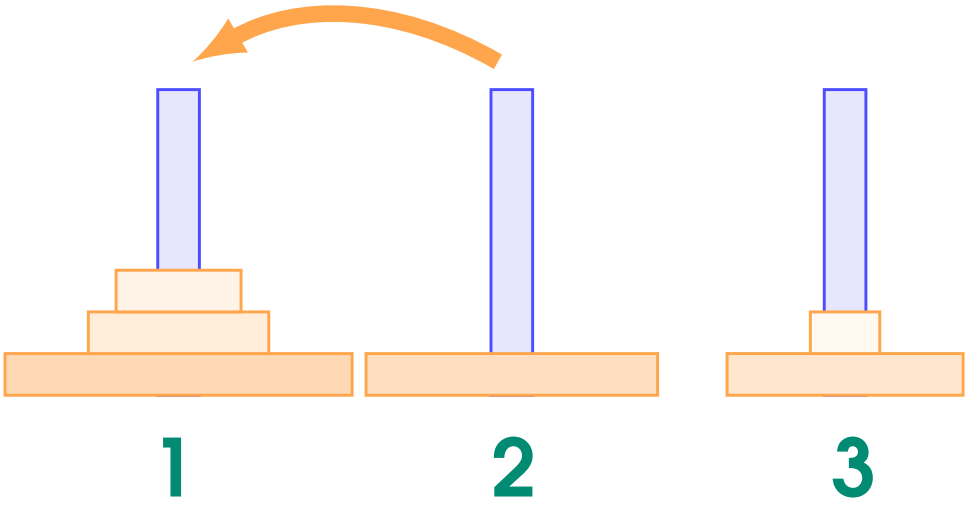

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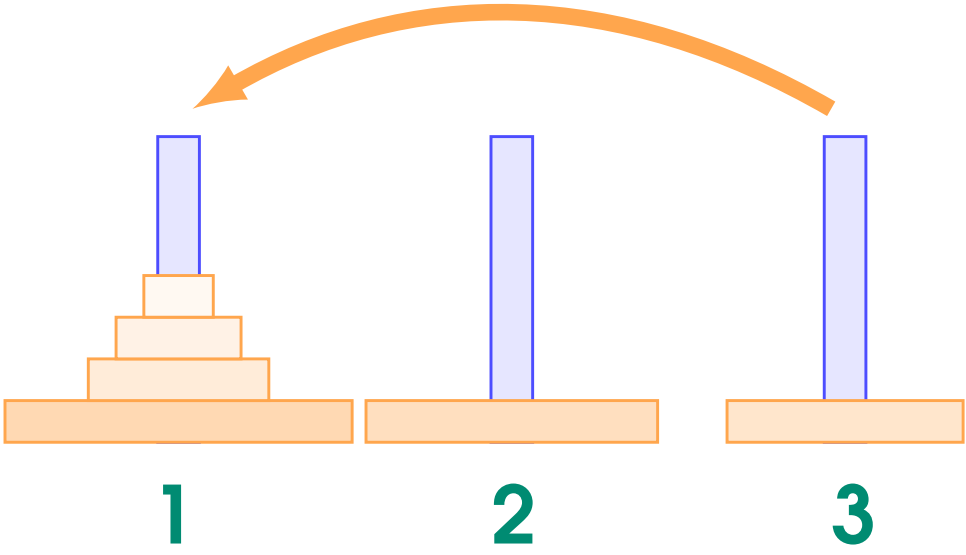

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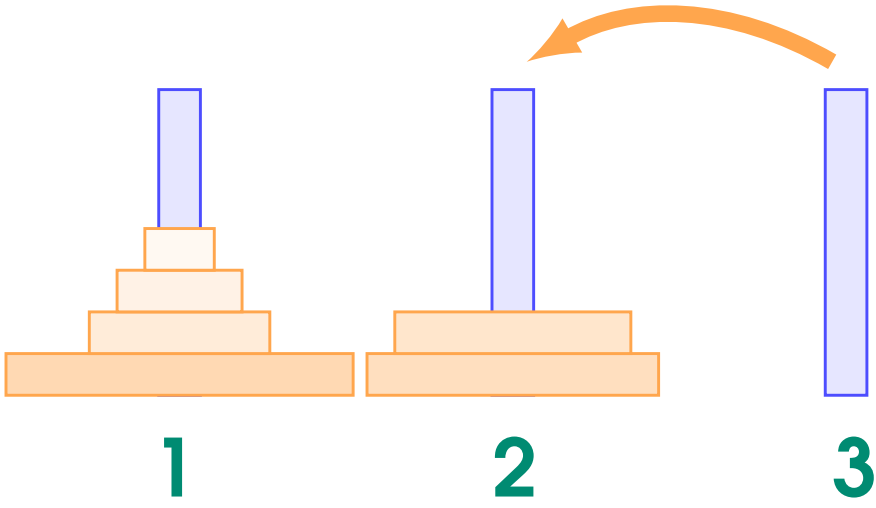

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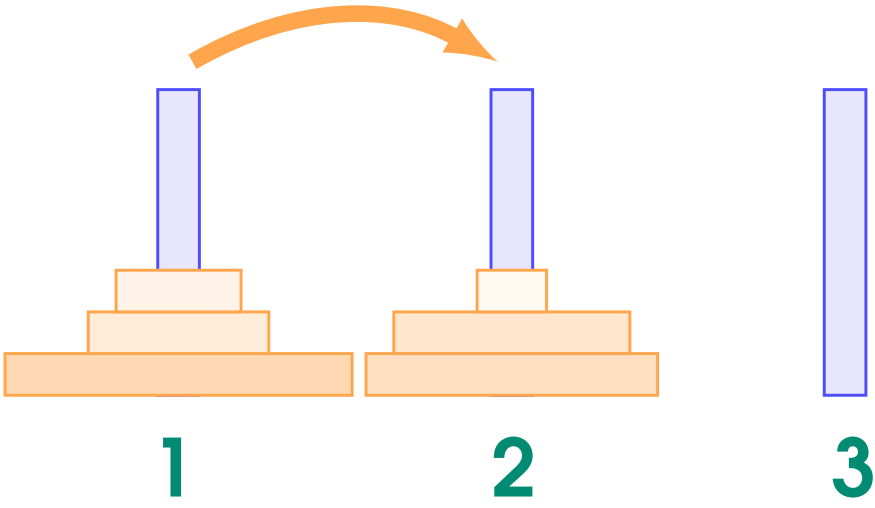

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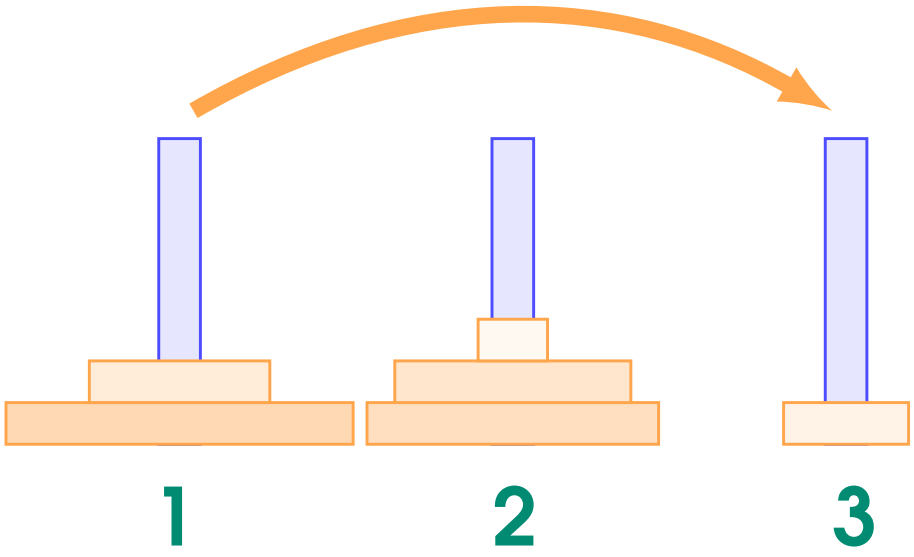

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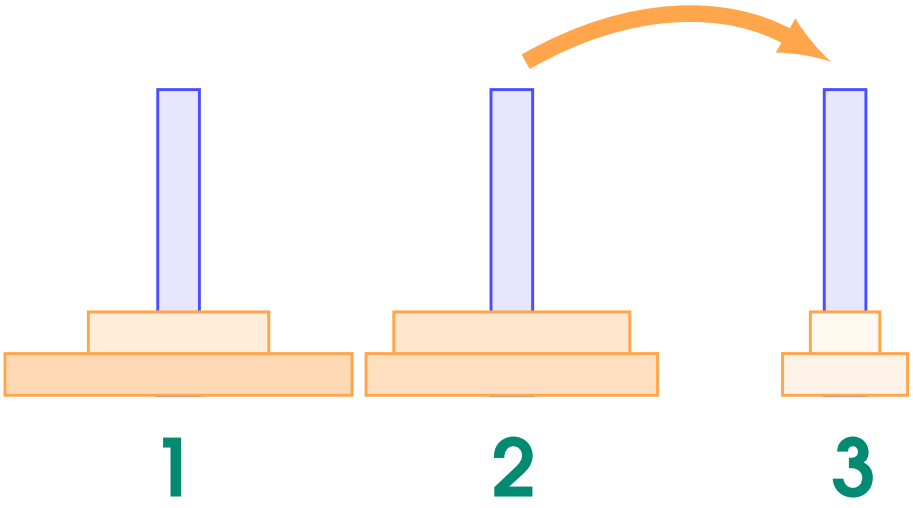

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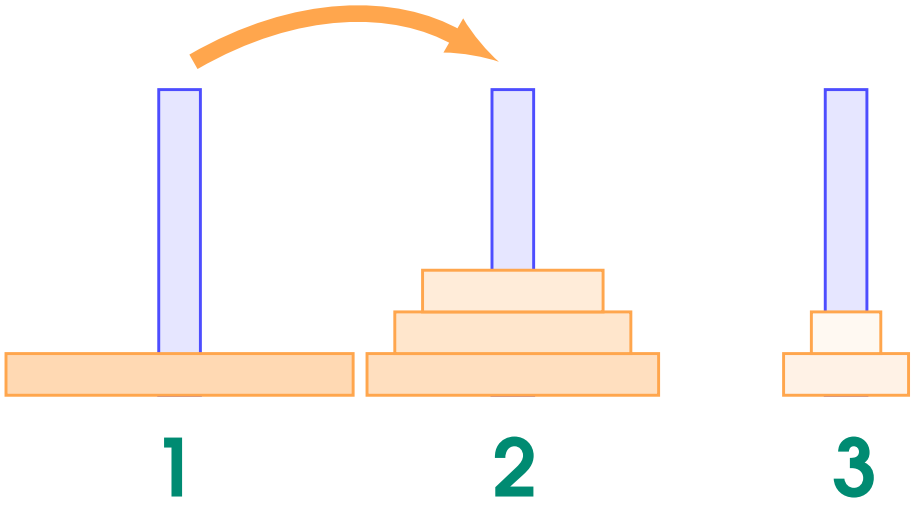

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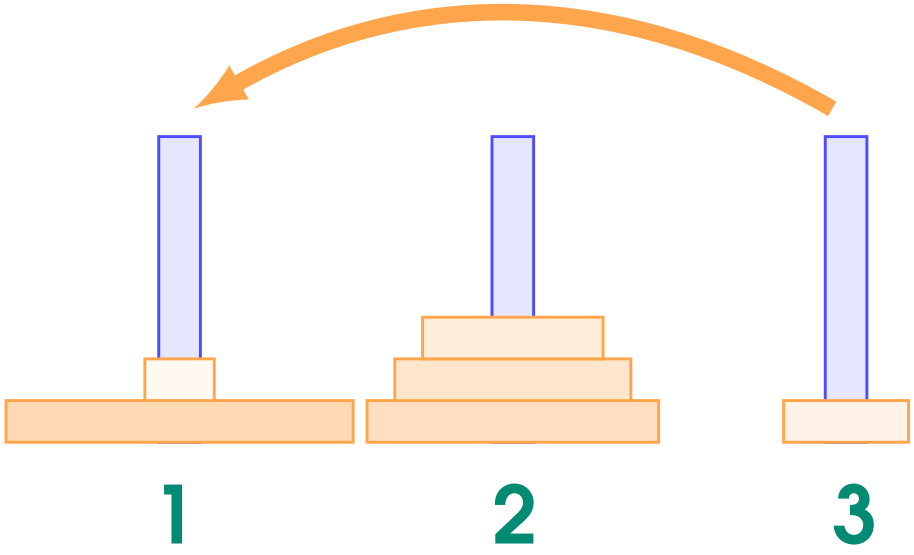

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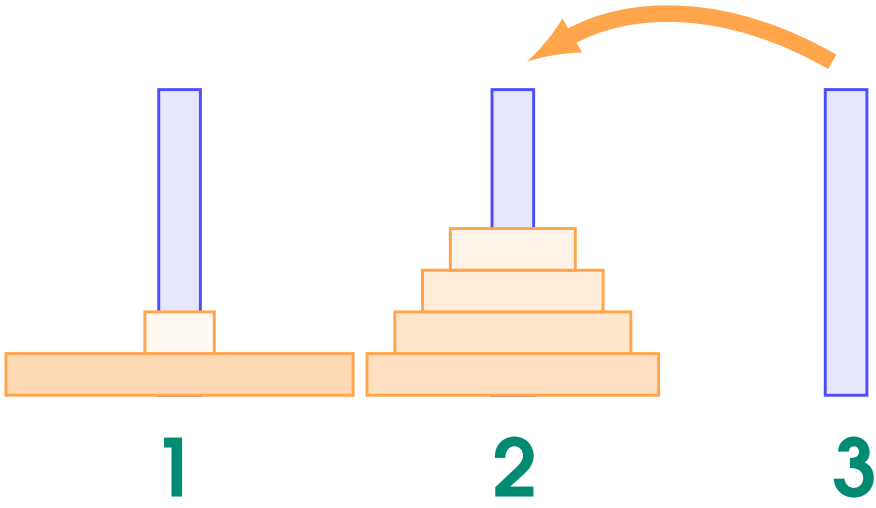

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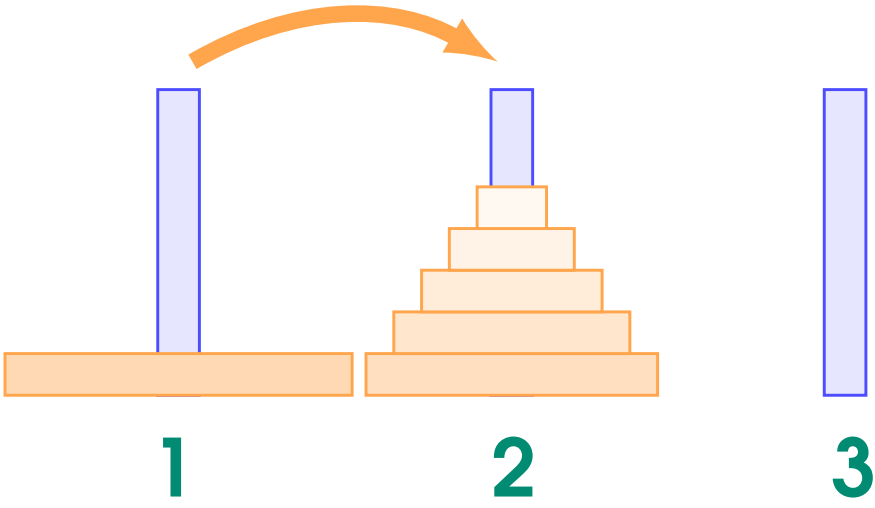

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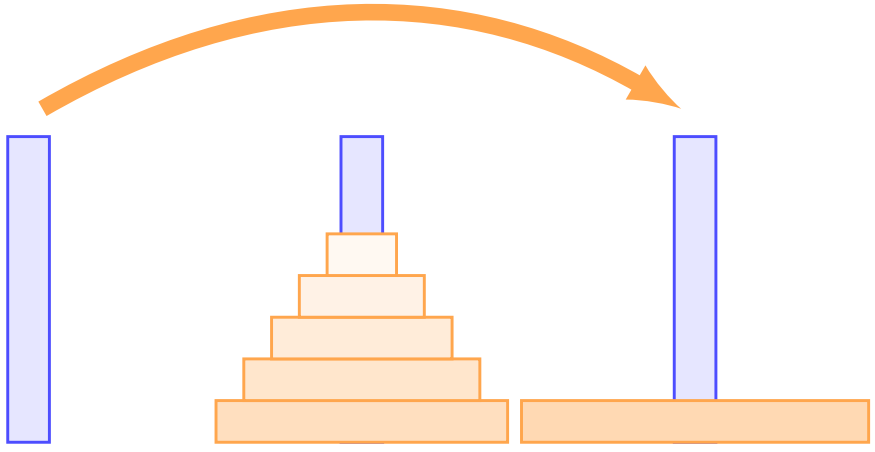

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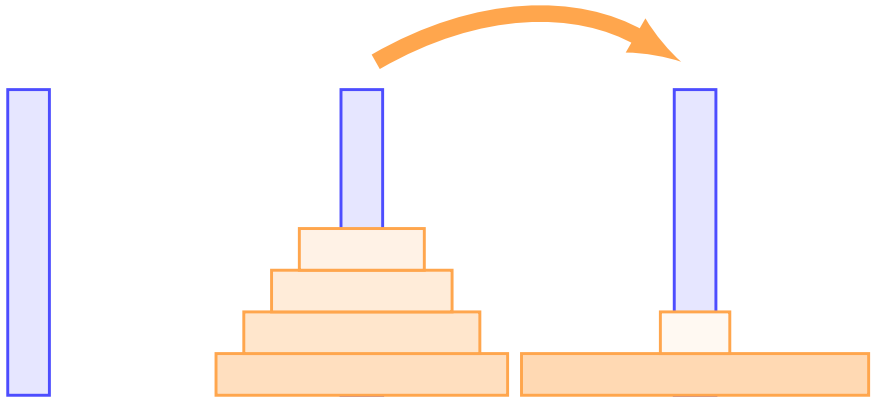

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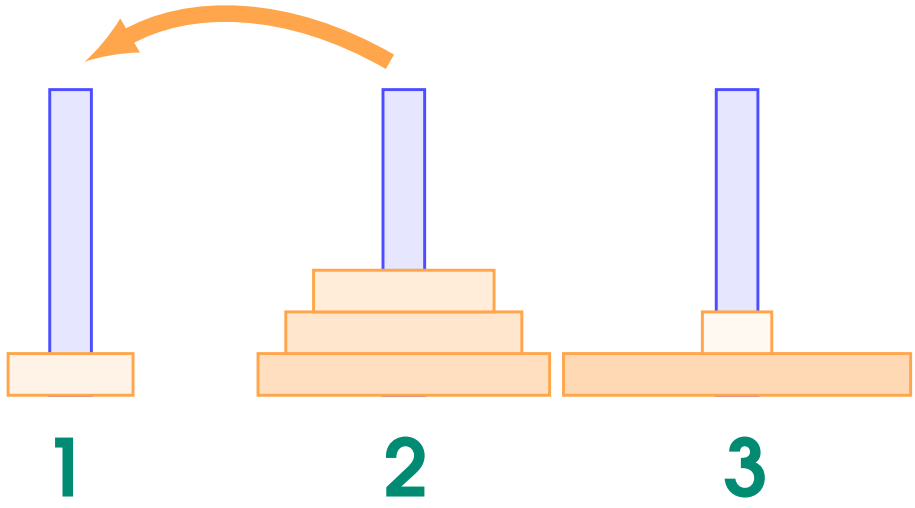

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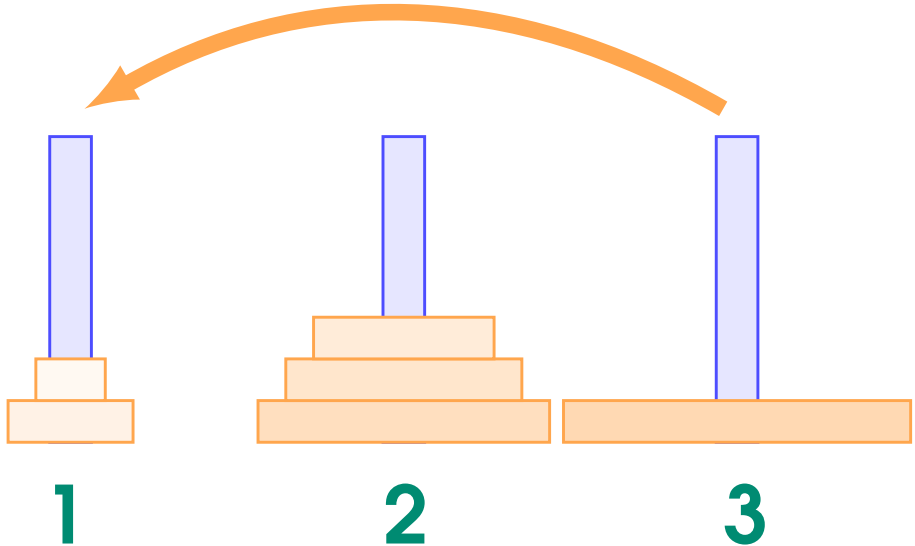

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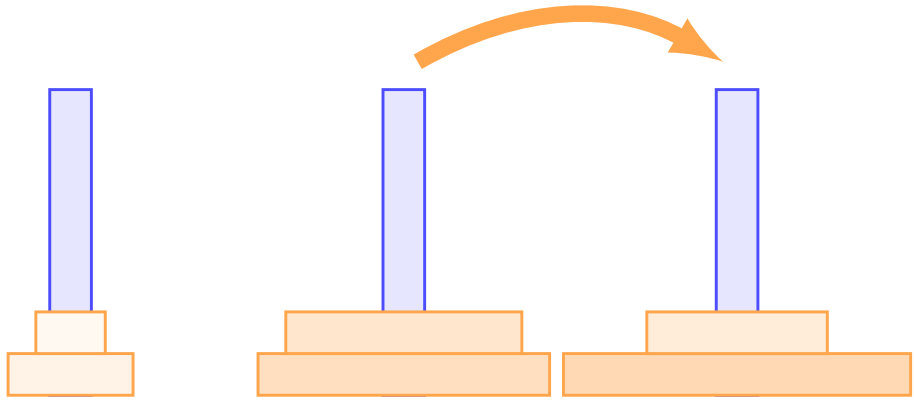

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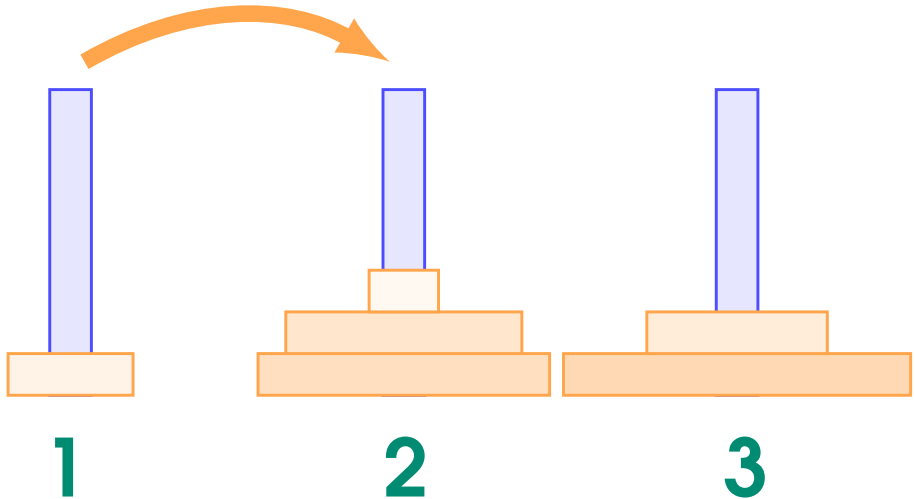

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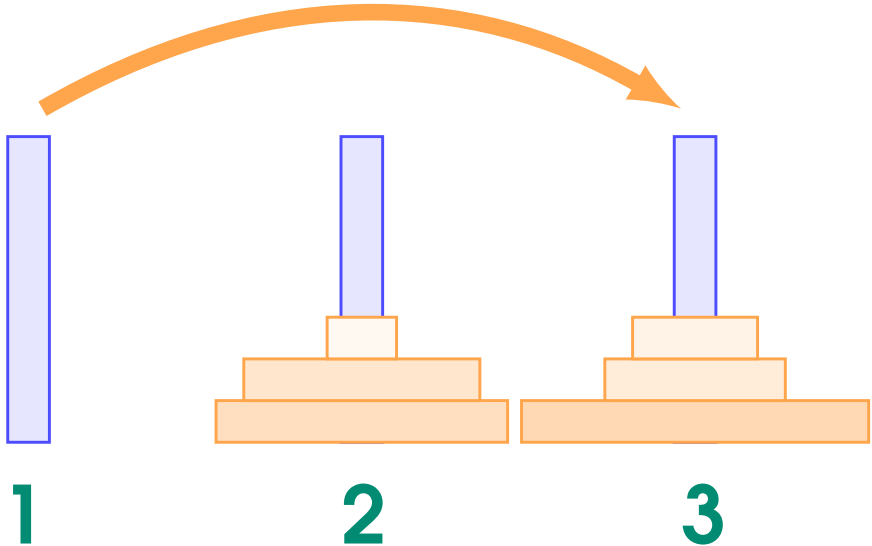

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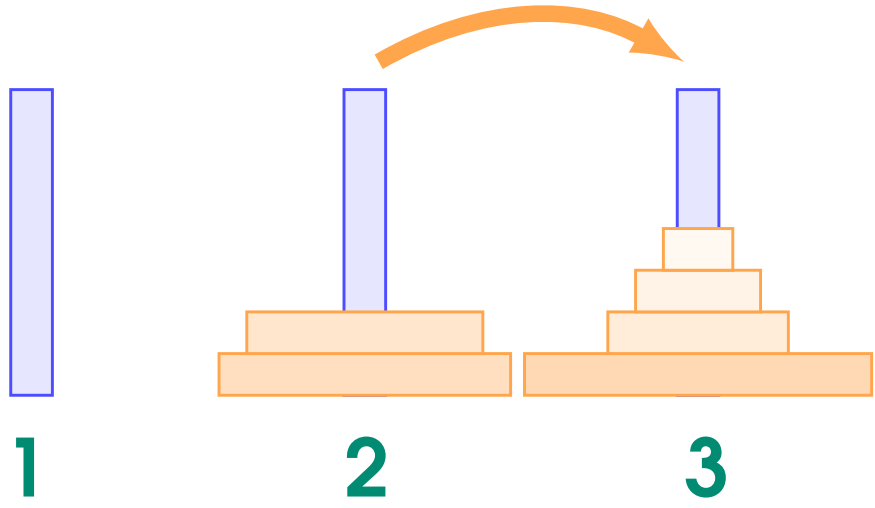

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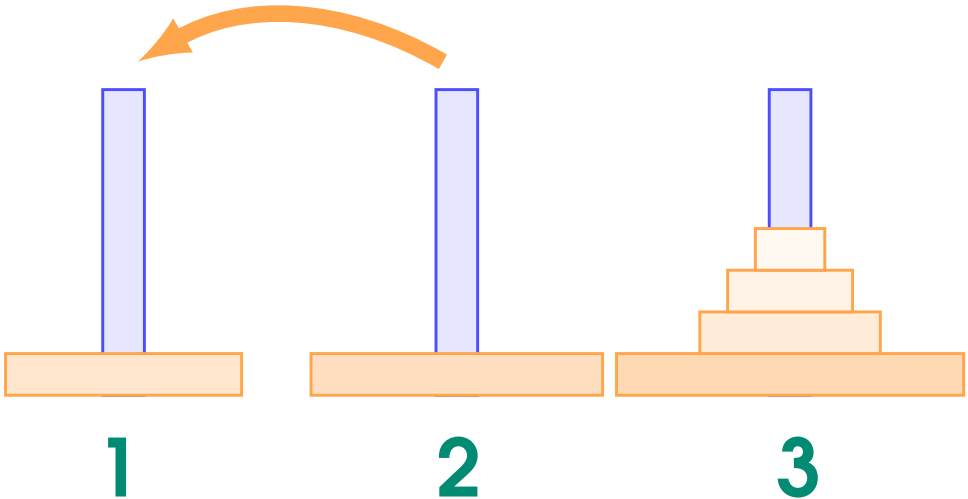

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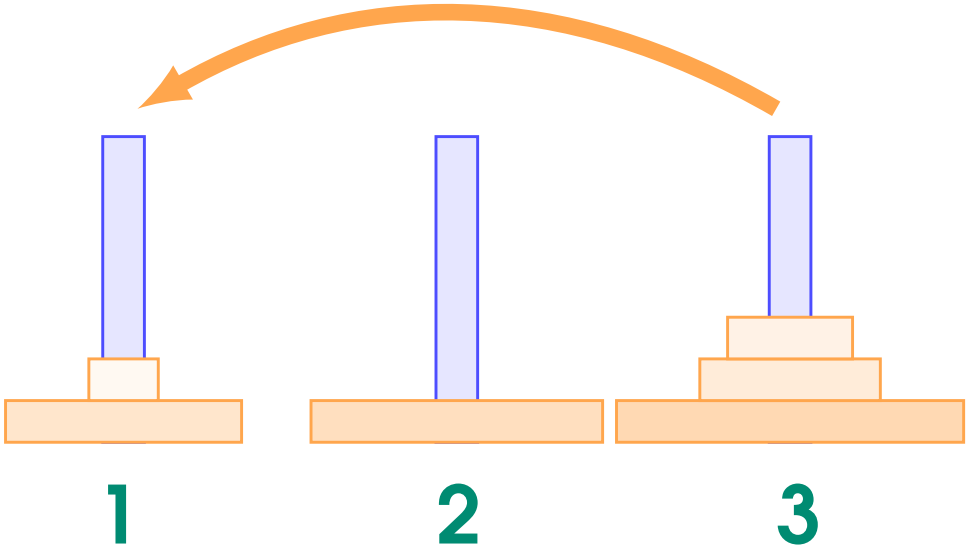

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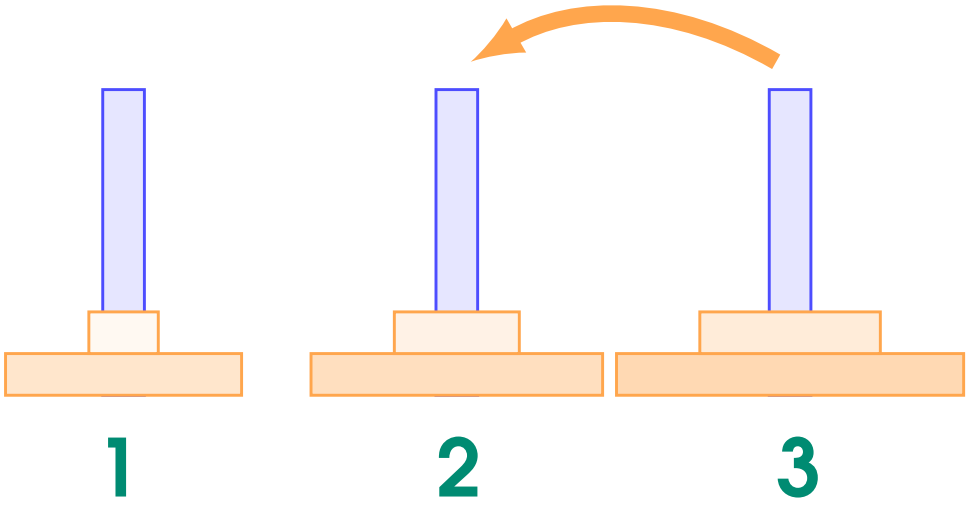

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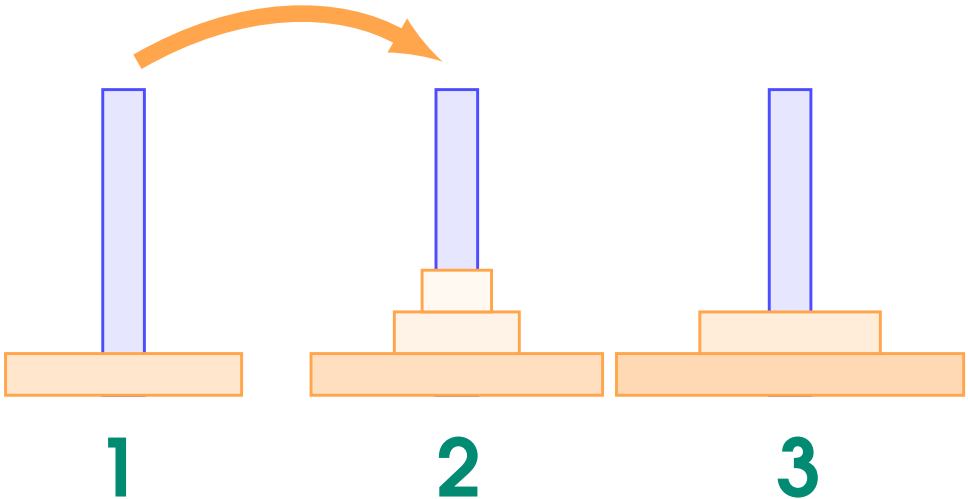

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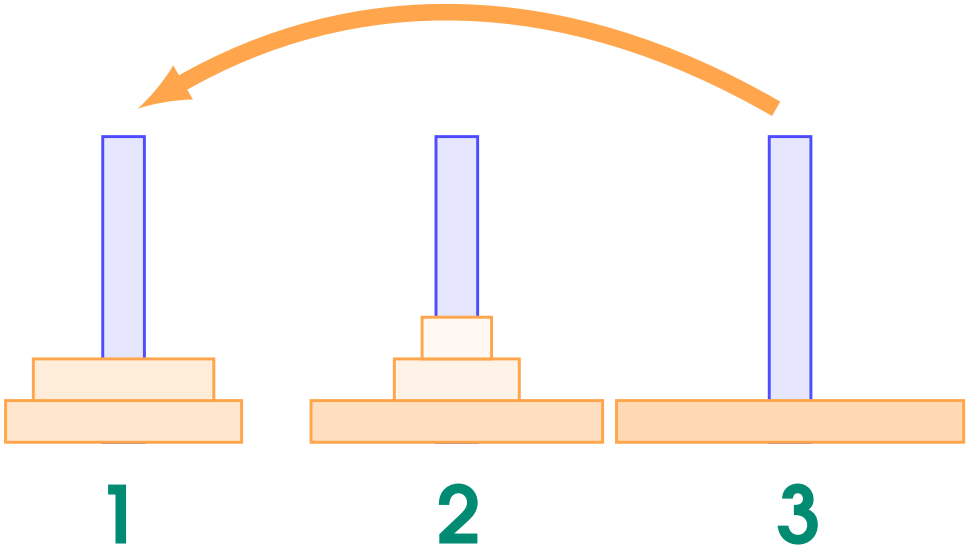

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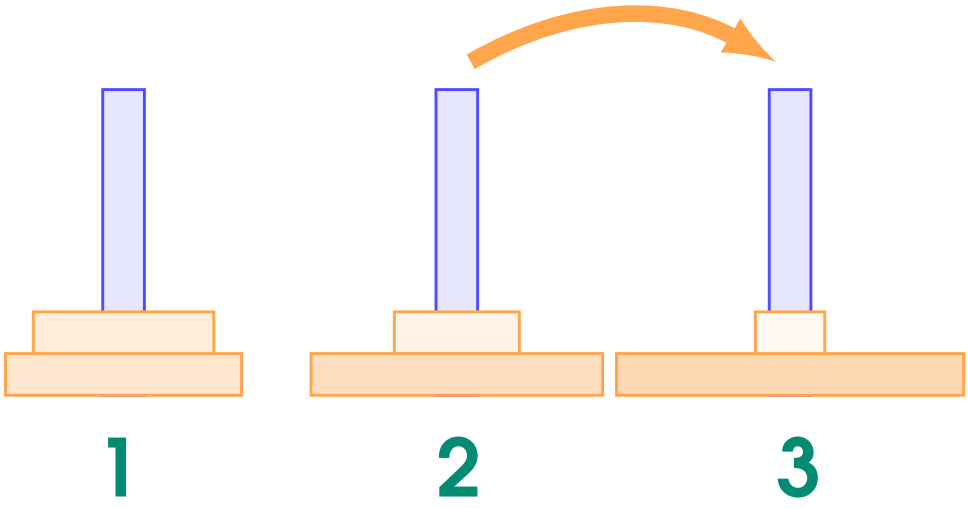

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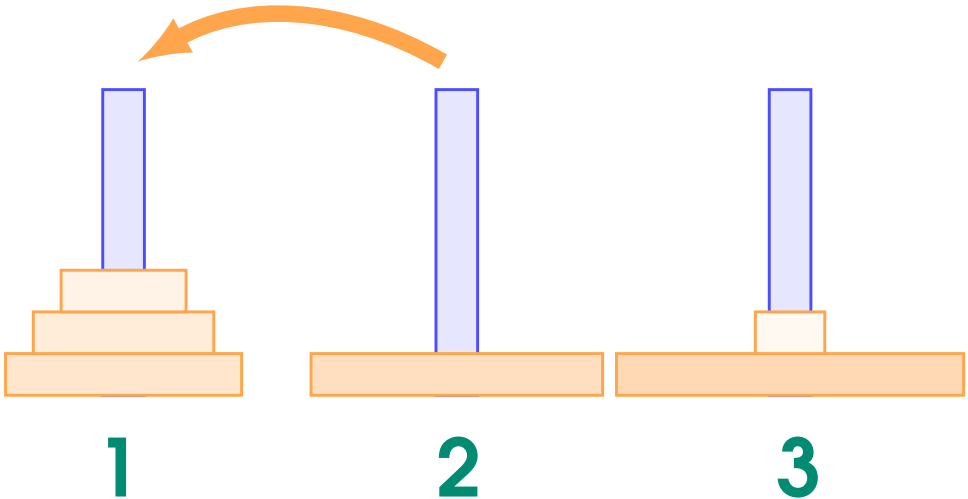

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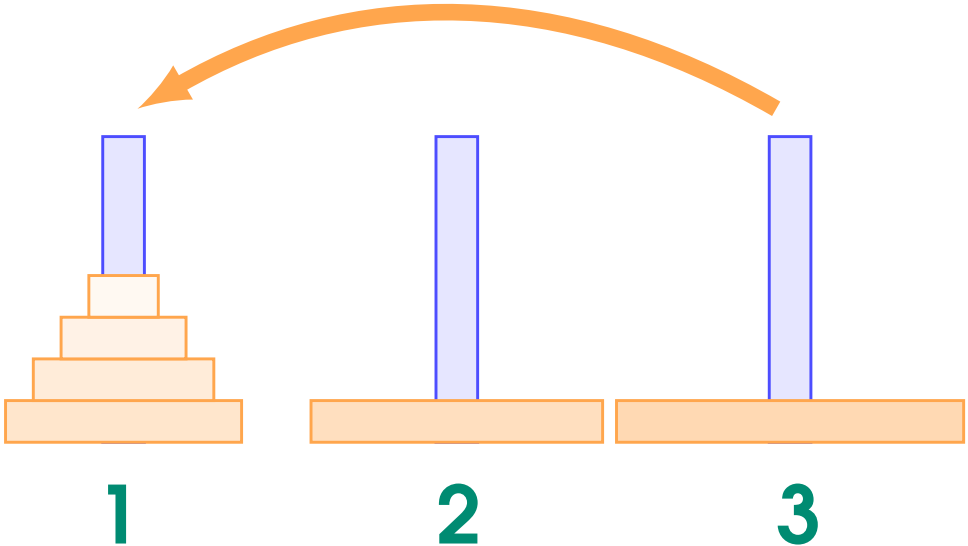

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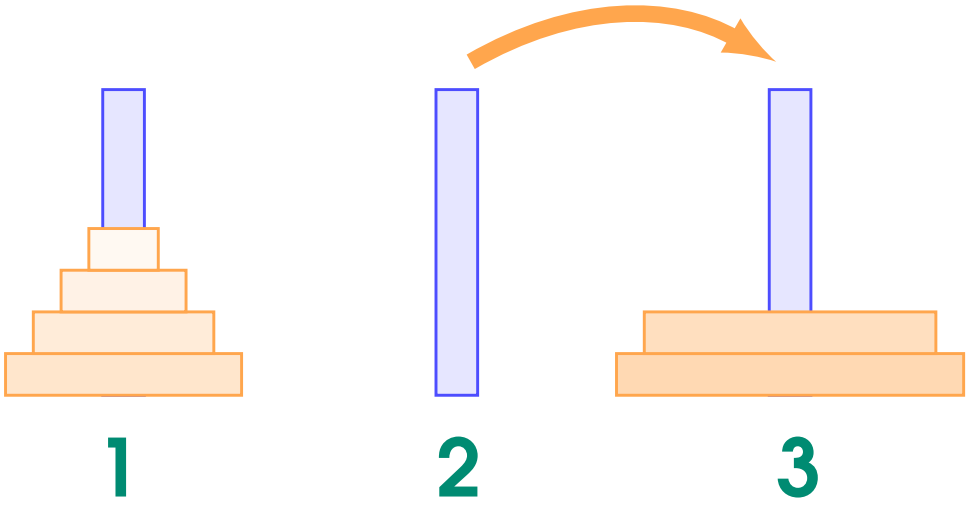

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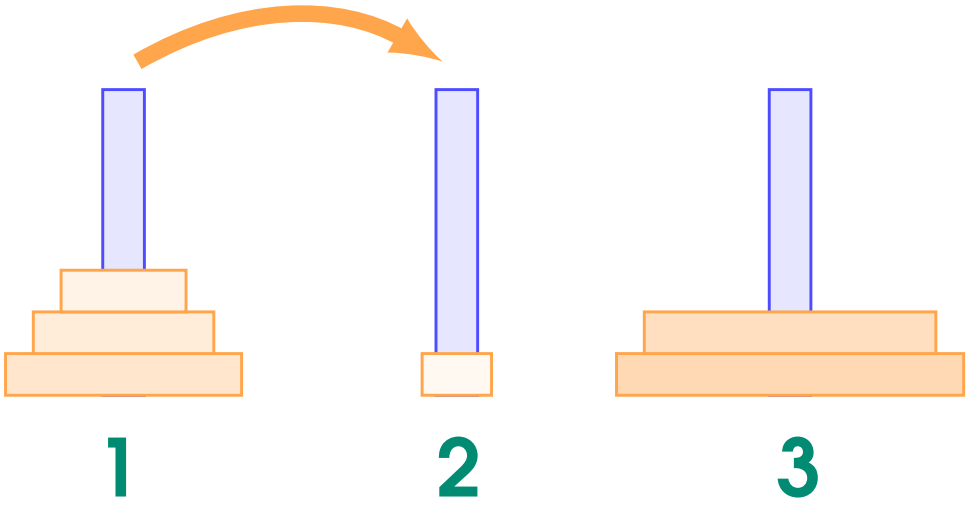

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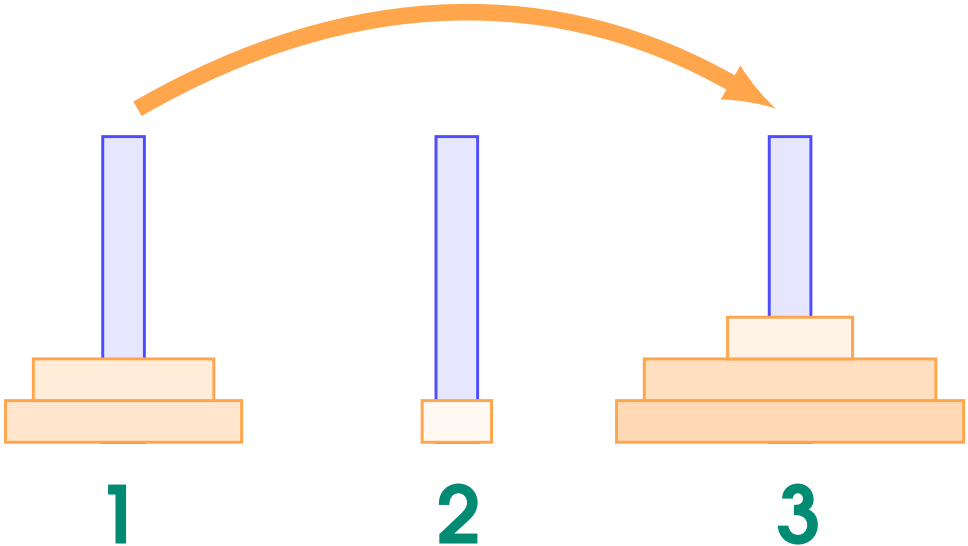

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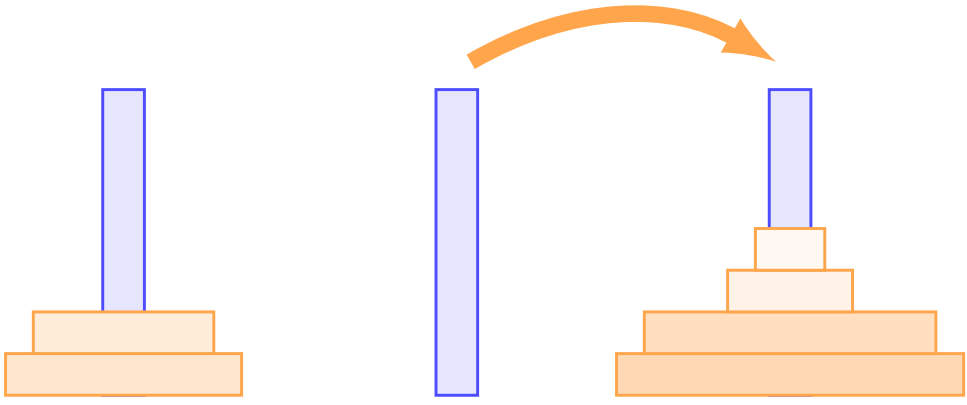
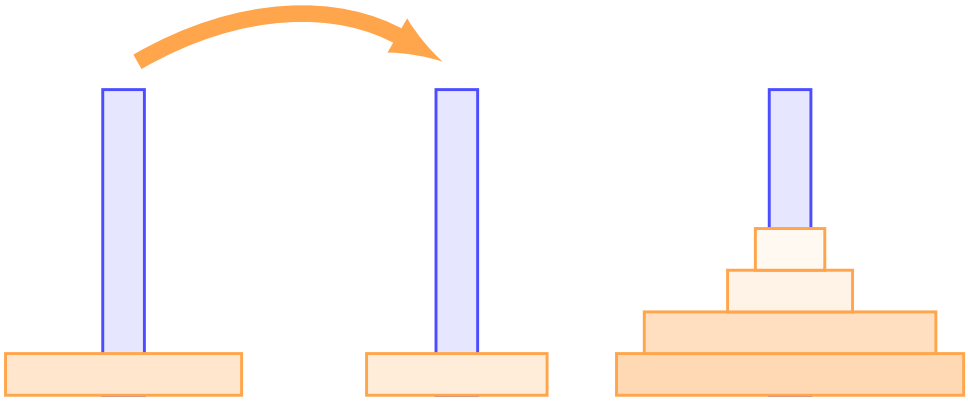


The Towers of Hanoi



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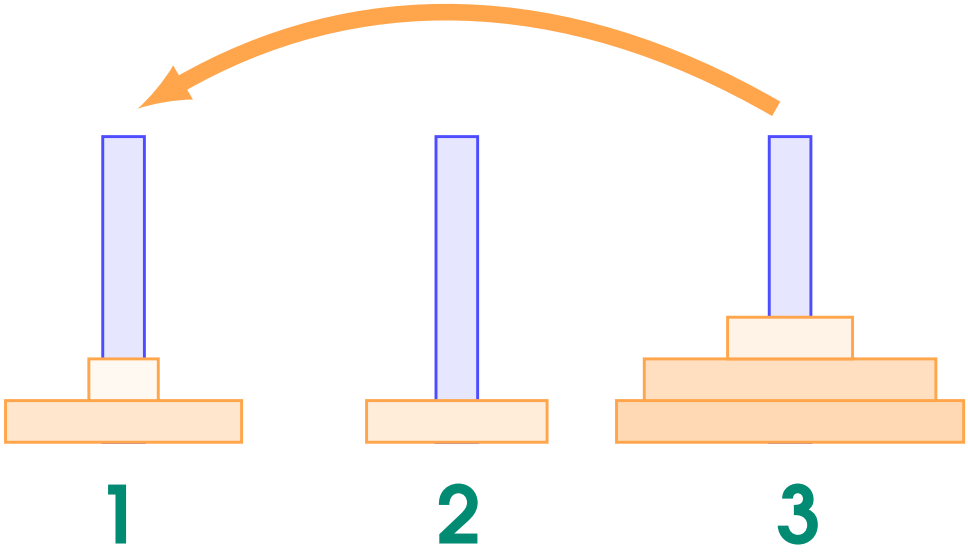

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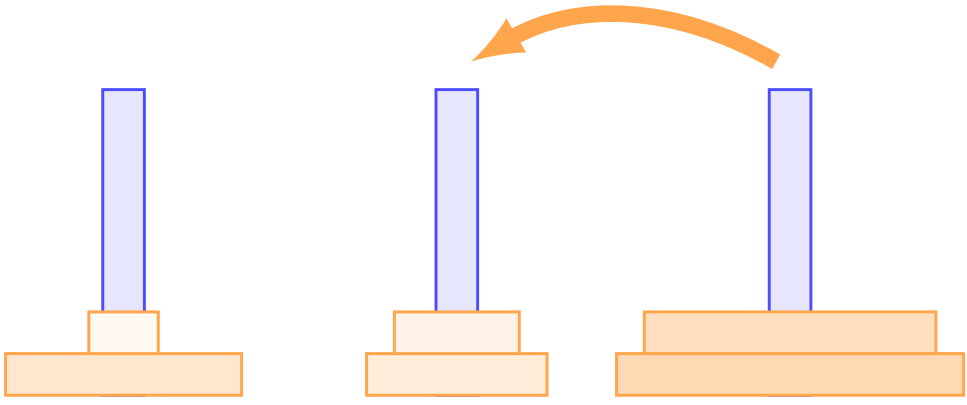

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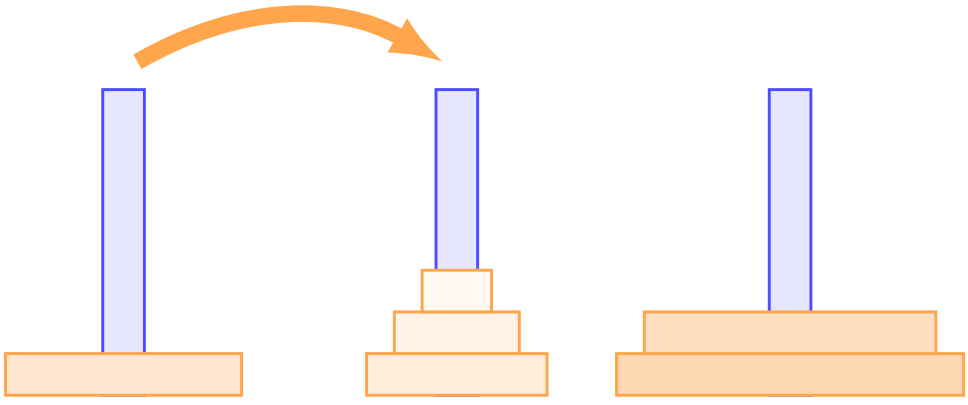

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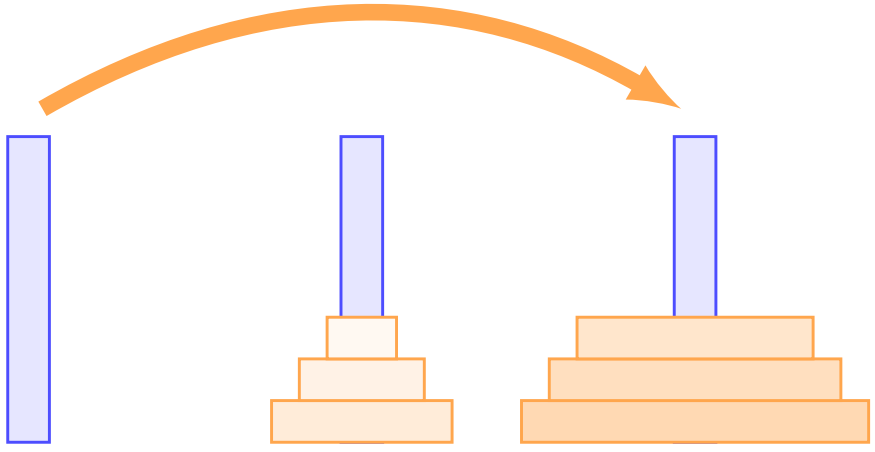

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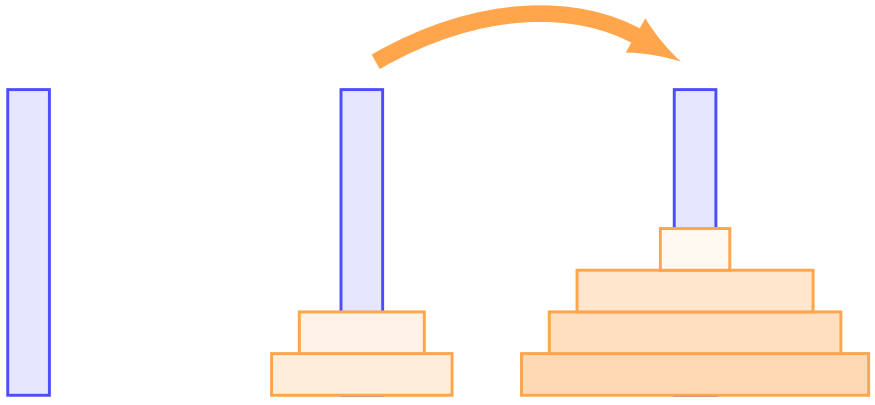

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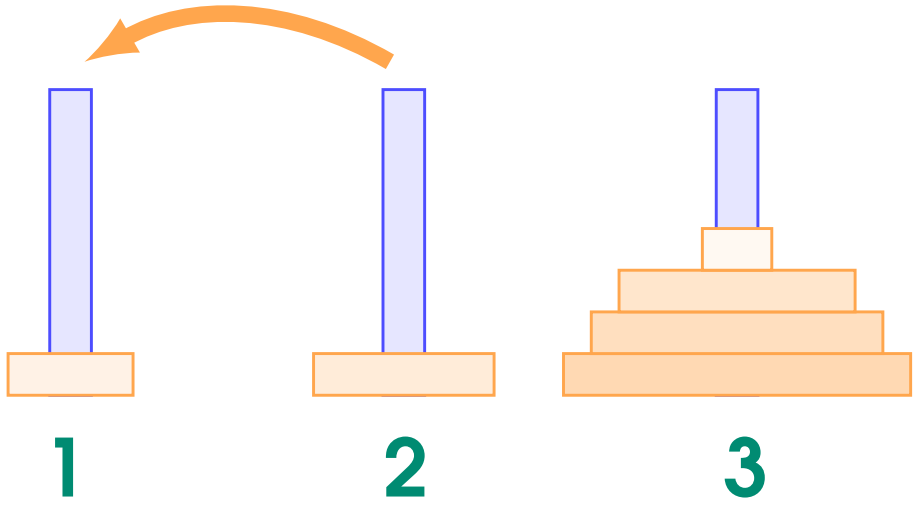

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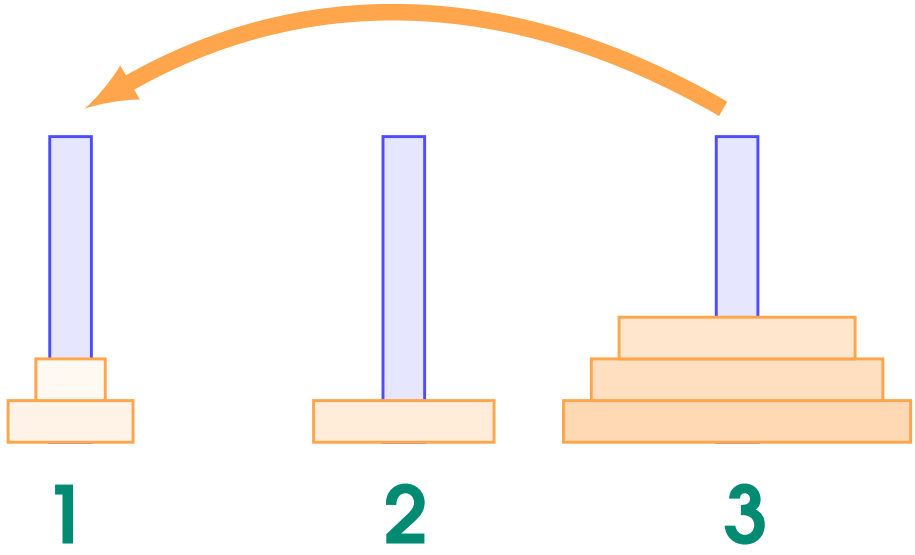

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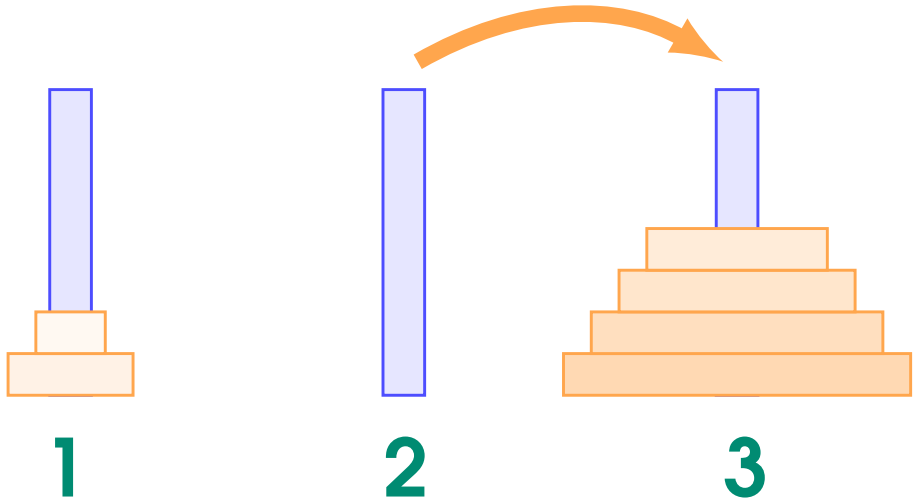

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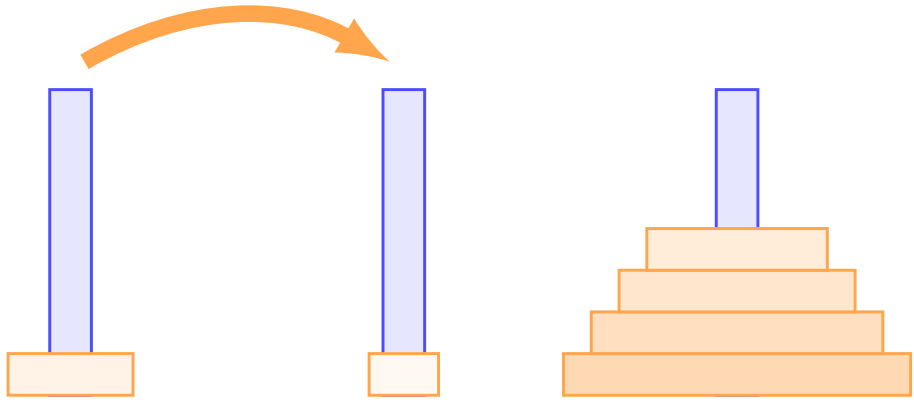

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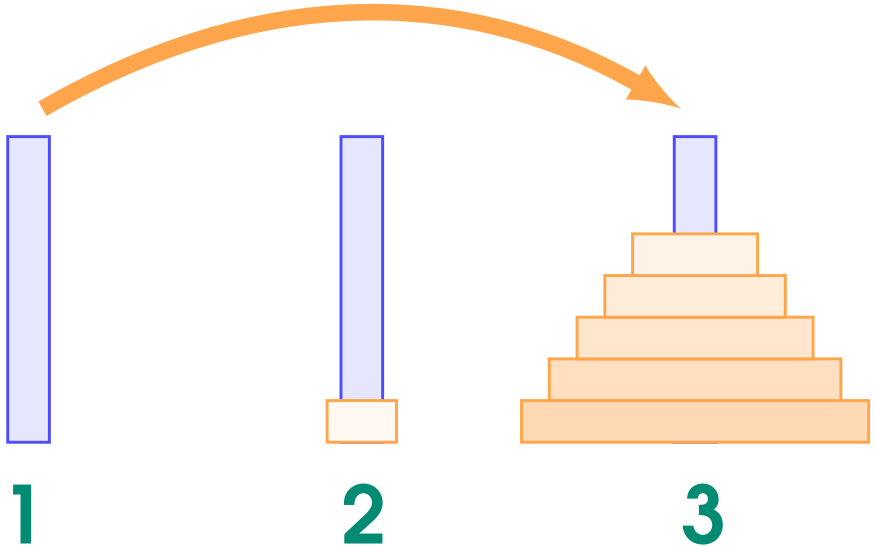

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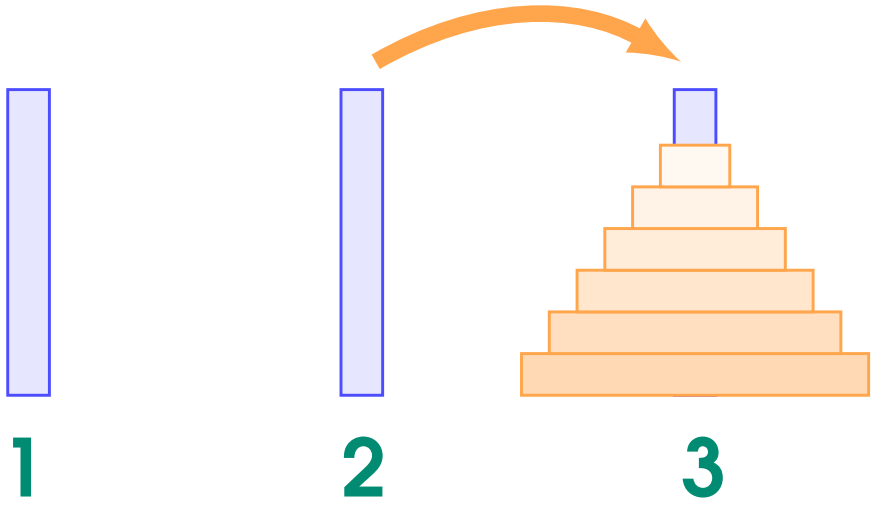


The Towers of Hanoi

Theorem The number of steps required for the MoveTower algorithm satisfies the following recurrence relation:

$$T(1) = 1, \qquad T(n) = 2T(n-1) + 1$$

for $n > 1$.

The Towers of Hanoi

MoveTower(n, 1, 3):

1. **If** $n = 1$ **then** Move(1, 3)
2. **Else**
   3. MoveTower($n - 1$, 1, 2)
   4. Move(1, 3)
   5. MoveTower($n - 1$, 2, 3)

$$T(n) = T(n-1) + 1 + T(n-1)$$

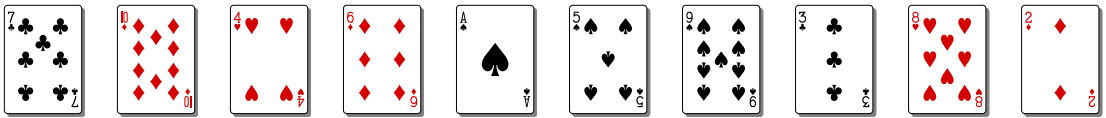
The Towers of Hanoi

n	1	2	3	4	5	6	7	8	$\cdots$
$T(n)$	1	3	7	15	31	63	127	255	$\cdots$

The Merge-Sort Algorithm

The Merge-Sort Algorithm

MERGESORT $([a_1, \ldots, a_n])$:



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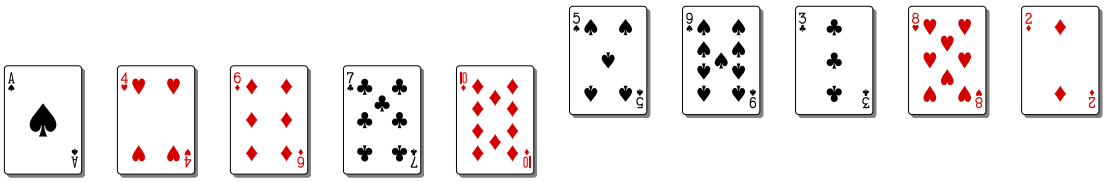

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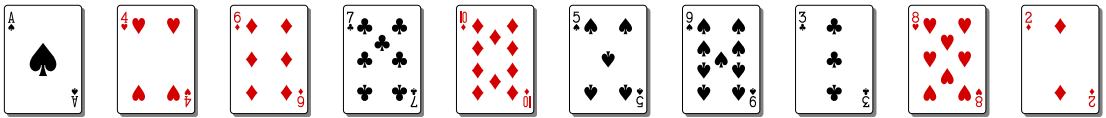

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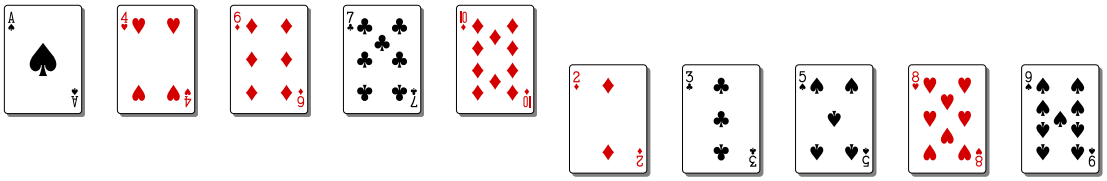

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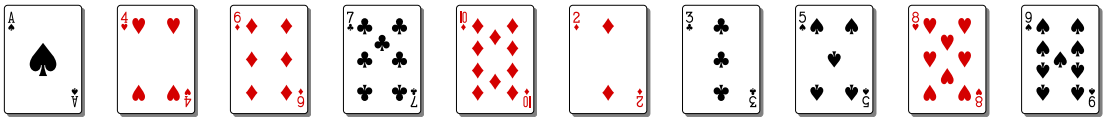

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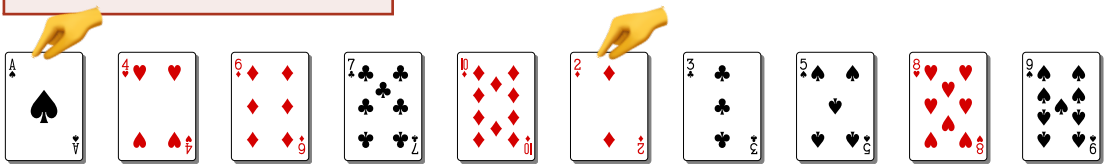

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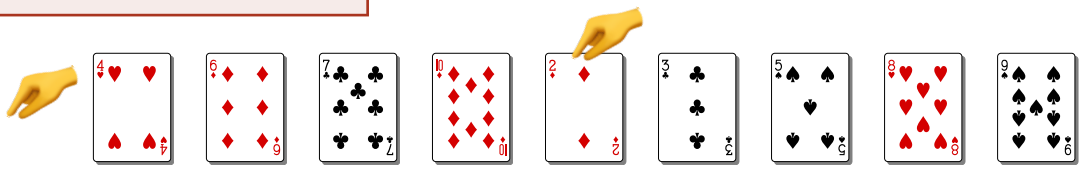

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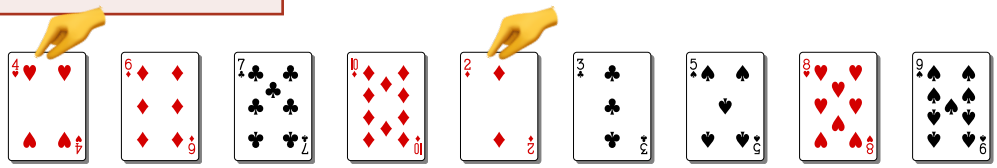

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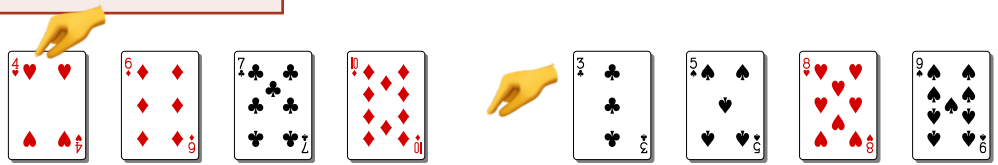

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The Merge-Sort Algorithm

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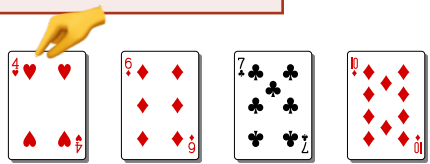

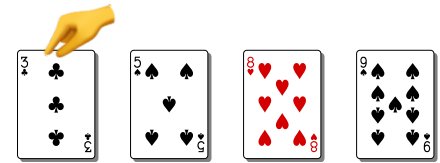

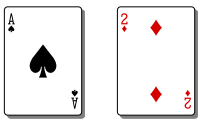

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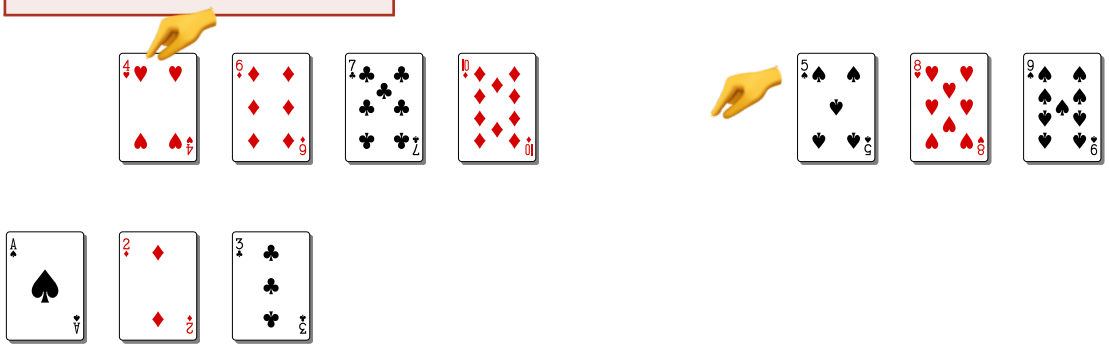

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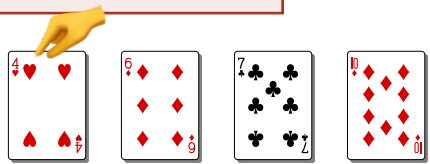

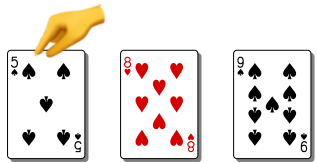

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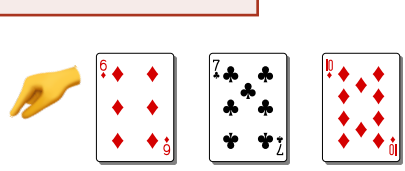

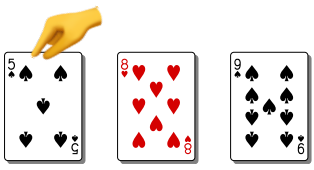

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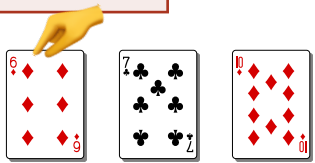

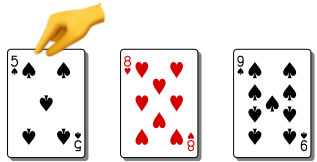

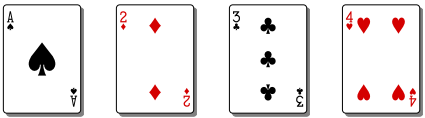

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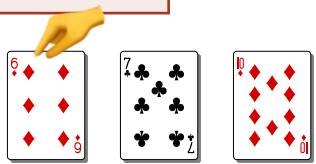

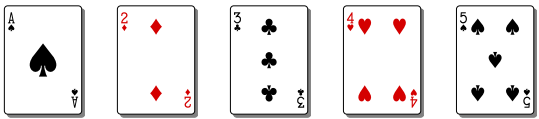

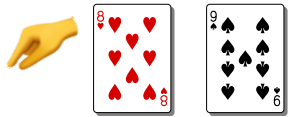

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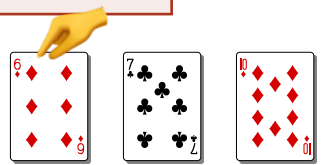

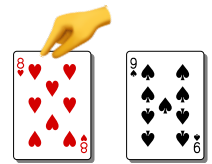

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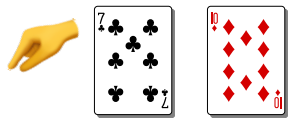

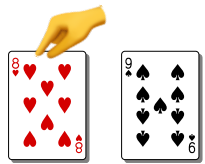

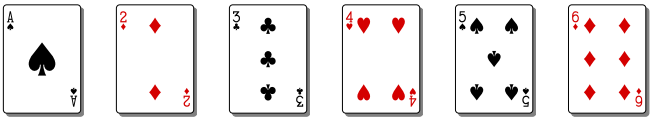

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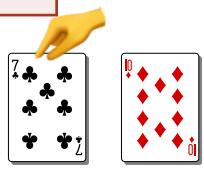

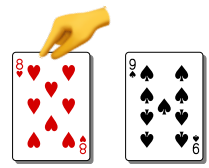

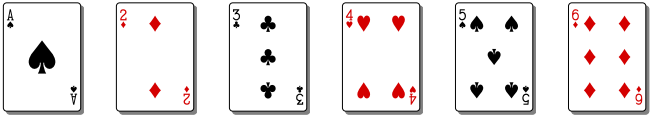

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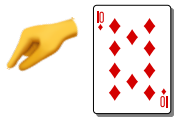

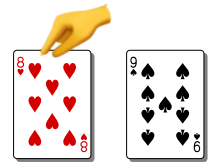

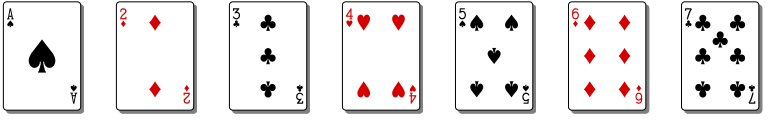

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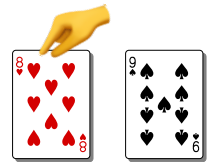

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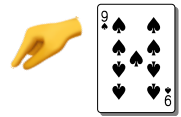

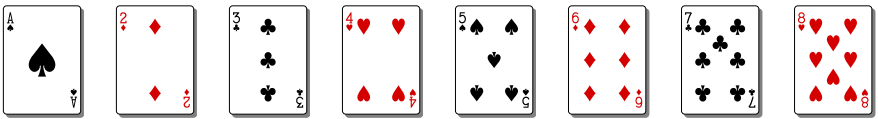

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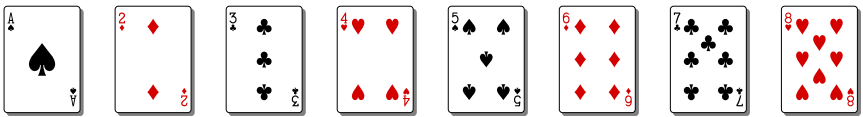

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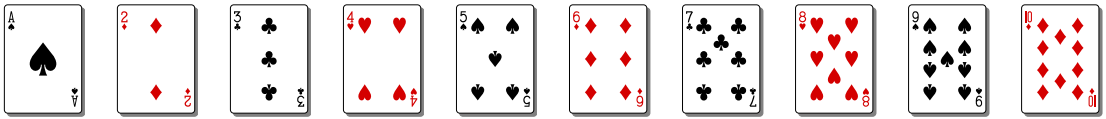


The Merge-Sort Algorithm

MERGESORT $([a_1, \ldots, a_n])$:

1. **If** $n = 1$ **then Return** $[a_1]$
2. **Else**
   3. $L \leftarrow$ MERGESORT $\left(\left[a_1, \ldots, a_{\lfloor n/2 \rfloor}\right]\right)$
   4. $R \leftarrow$ MERGESORT $\left(\left[a_{\lfloor n/2 \rfloor + 1}, \ldots, a_n\right]\right)$
   5. $A \leftarrow$ MERGE(L, R)
   6. **Return** A

The Merge-Sort Algorithm

Theorem The number of steps required for the MERGESORT algorithm satisfies the following recurrence relation:

$$T(1) = 1, \qquad T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right) + n$$

for $n > 1$.

The Merge-Sort Algorithm

Theorem The number of steps required for the MERGESORT algorithm satisfies the following recurrence relation:

$$T(1) = 1, \qquad T(n) \approx 2T\left(\left\lceil \frac{n}{2} \right\rceil\right) + n$$

for $n > 1$.

The Merge-Sort Algorithm

MERGESORT $([a_1, \ldots, a_n])$:

1. **If** $n = 1$ **then Return** $[a_1]$
2. **Else**
3. $L \leftarrow$ MERGESORT $([a_1, \ldots, a_{\lfloor n/2 \rfloor}])$
4. $R \leftarrow$ MERGESORT $([a_{\lfloor n/2 \rfloor + 1}, \ldots, a_n])$
5. $A \leftarrow$ MERGE(L, R)
6. **Return** A

$$T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + T\left(\left\lceil \frac{n}{2} \right\rceil\right) + n$$

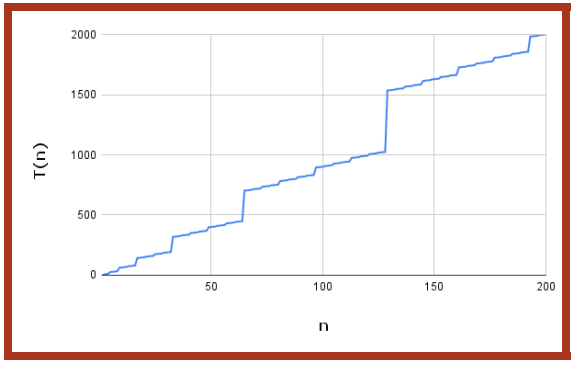
The Merge-Sort Algorithm

n	1	2	3	4	5	6	7	8	$\cdots$
$T(n)$	1	4	11	12	27	28	31	32	$\cdots$

The Merge-Sort Algorithm

B3 | fx | =2*VLOOKUP(ceiling(A3/2),A:B,2)+A3

	A	B	C	D
1	n	T(n)		
2	1	1		
3	2	4		
4	3	11		
5	4	12		
6	5	27		
7	6	28		
8	7	31		
9	8	32		
10	9	63		
11	10	64		
12	11	67		
13	12	68		
14	13	75		
15	14	76		
16	15	79		
17	16	80		
18	17	143		
19	18	144		



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