5CCS2FC2: Foundations of Computing II

Introduction to Graph Algorithms

Week 4

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Graph Algorithms

Graph Algorithms

- We expand the notation for the **termination time** for algorithms on graphs

$$T(|V|, |E|) = \text{The number of } \textit{primitive operations} \text{ on a graph with } |V| \text{ verticies and } |E| \text{ edges}$$

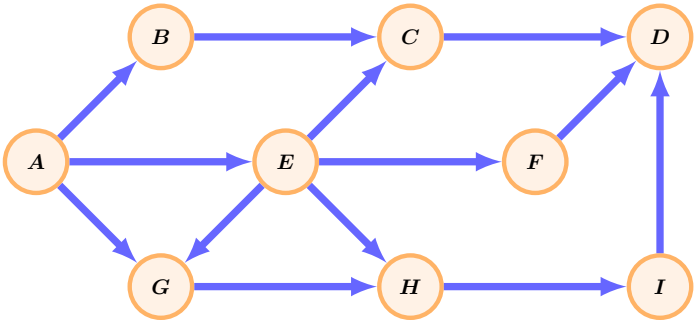
- Our **primitive operations** are operations we can perform in $O(1)$ steps.

Breadth-First-Search

BREDTH-FIRST-SEARCH:

1. Select root vertex and add to a queue
2. **While** queue is not-empty:
    3. Inspect the first element u on the queue.
    4. Add any unvisited neighbours of u to the queue.
    5. Remove u from the queue.

Breadth-First-Search



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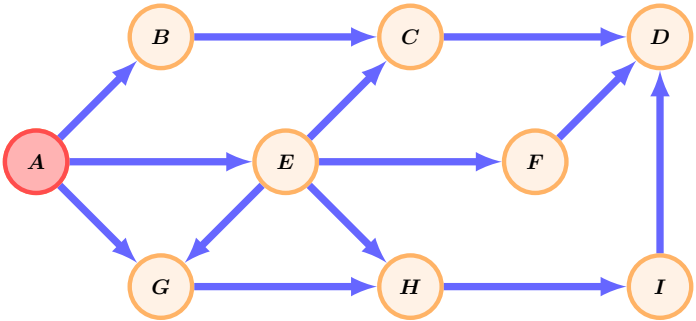

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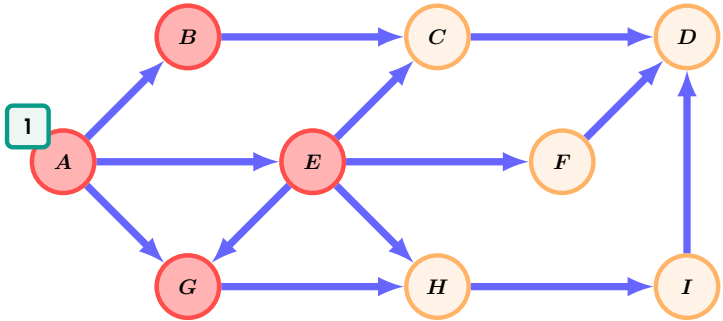
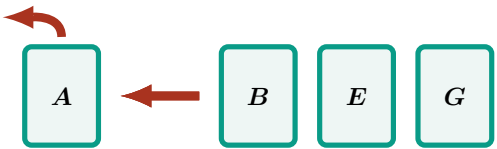
Breadth-First-Search





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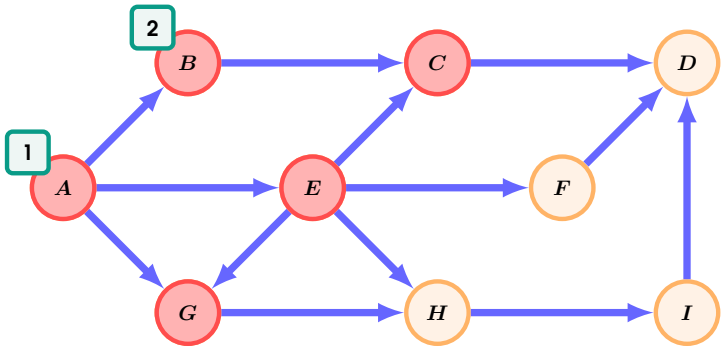

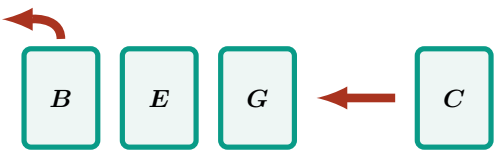

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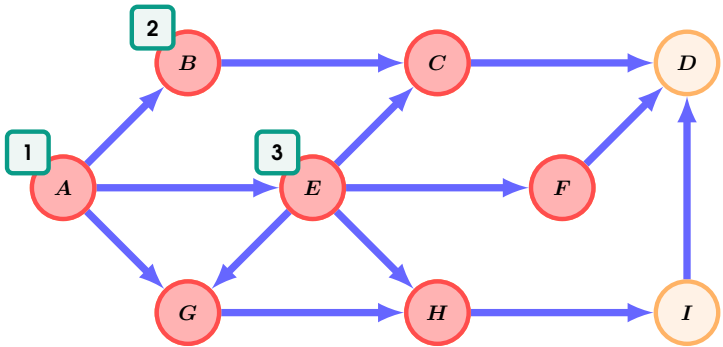

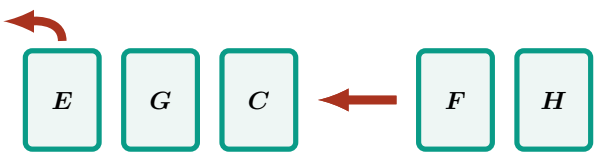

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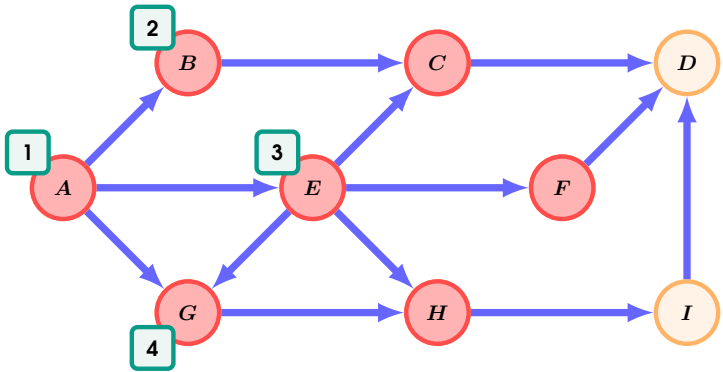

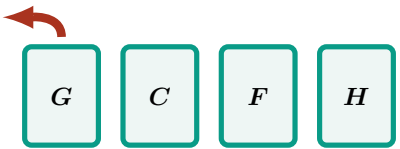

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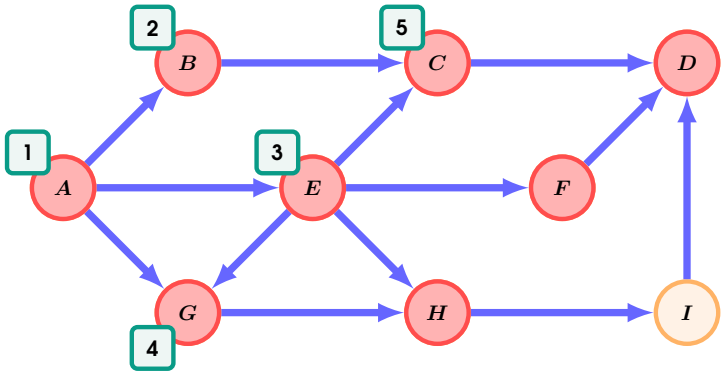

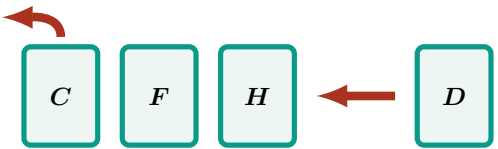

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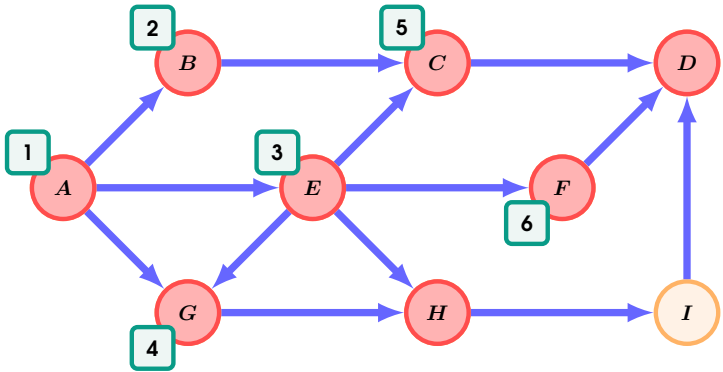

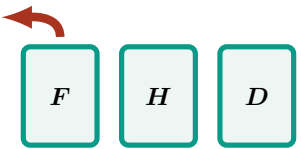

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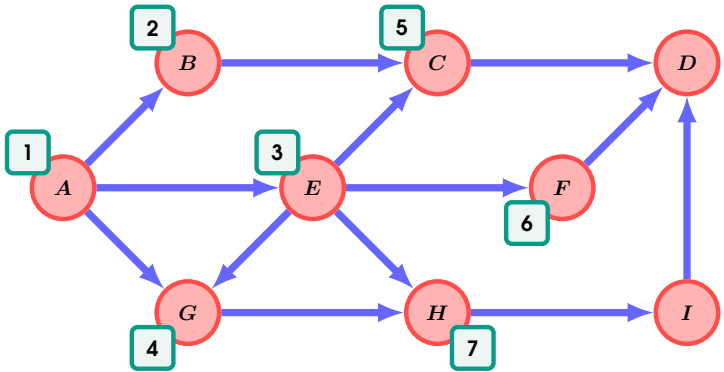

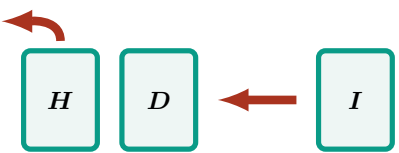

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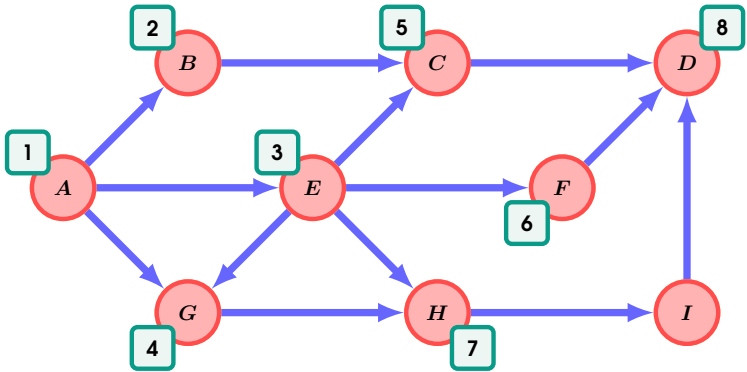

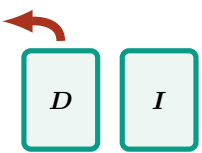

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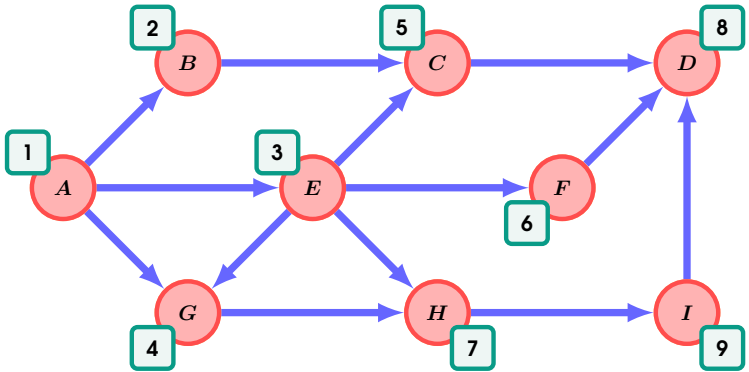

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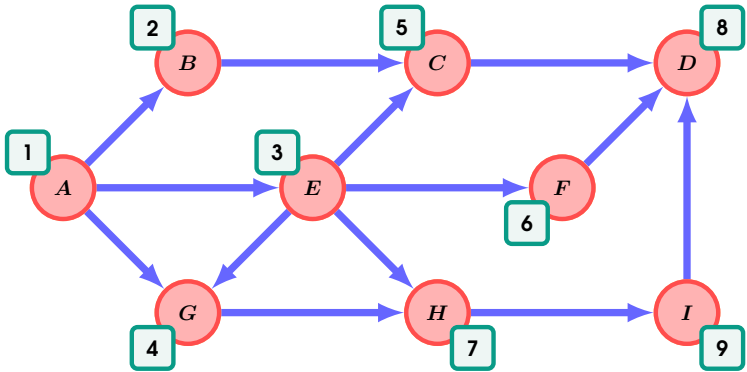


Queue Empty so Terminate!

Breadth-First-Search

Theorem The termination time for Breadth-First-Search is $O(|V| + |E|)$.

Proof:

- Note that every vertex is **enqueued** and **dequeued** at most once.
  So **Steps 3–5** are repeated once for each $u \in V$.
- Inspecting (**Step 3**) and Adding (**Step 5**) each take at most $O(1)$ operations.
- However, **Step 4** requires $O(|\mathbf{Adj}(u)|)$ operations to queue each of the adjacent vertices to u.

(where $\mathbf{Adj}(u)$ denotes the set of verticies adjacent to u)

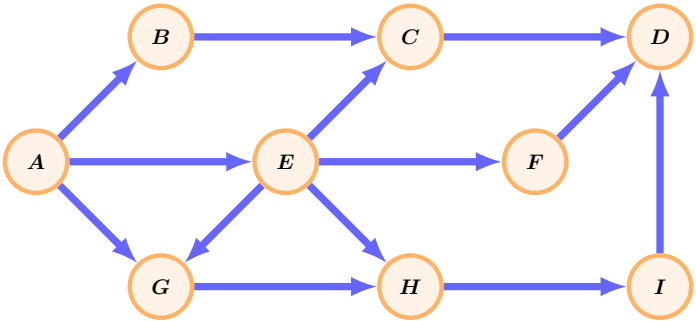
- Altogether we have

Depth-First-Search

DEPTH-FIRST-SEARCH:

1. Select root vertex and add to a stack
2. **While** stack is not-empty:
    3. Inspect the top element u on the stack.
    4. Add any unvisited neighbours of u to the stack.
    5. Remove u from the stack.

Depth-First-Search



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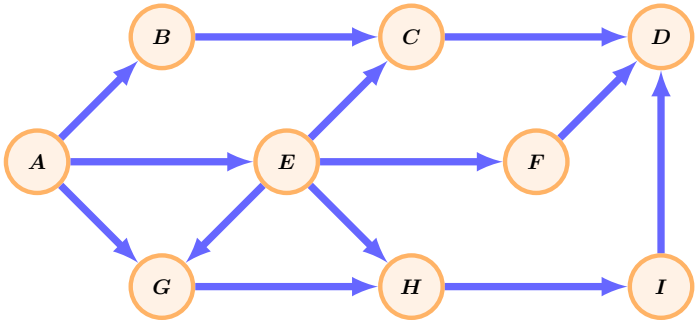

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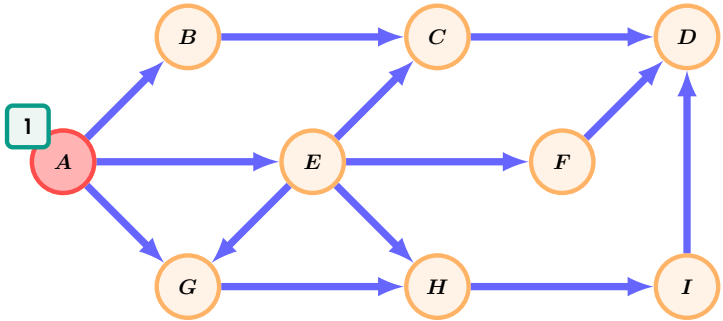

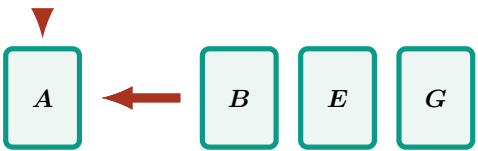

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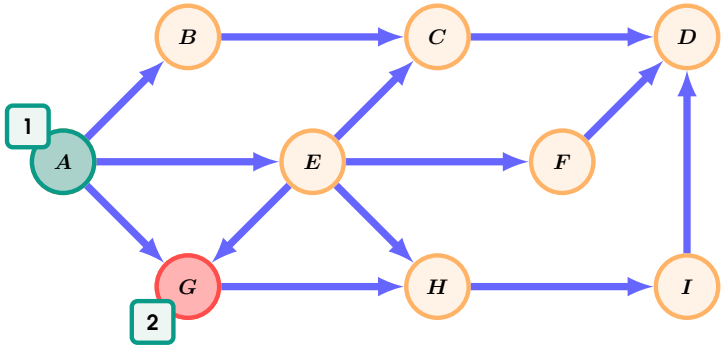

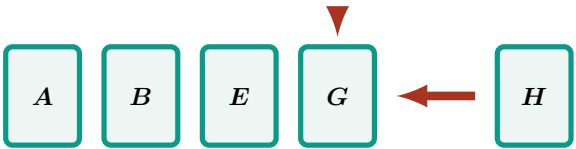

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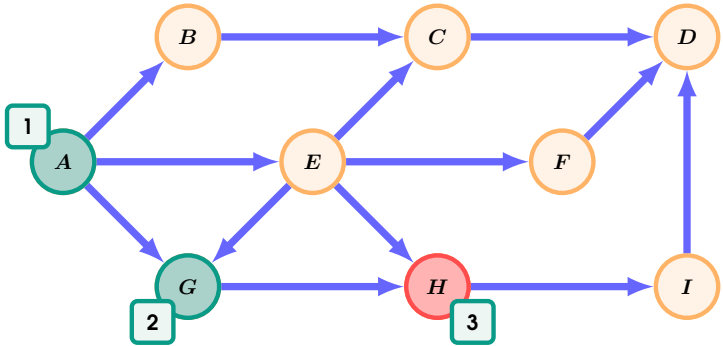

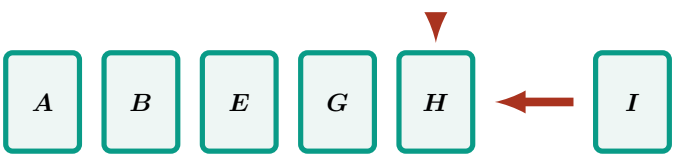

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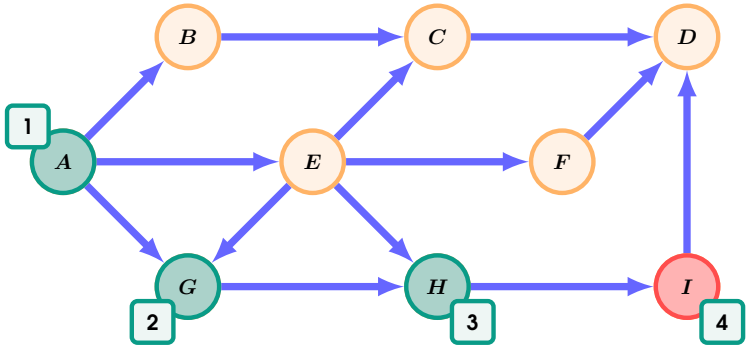

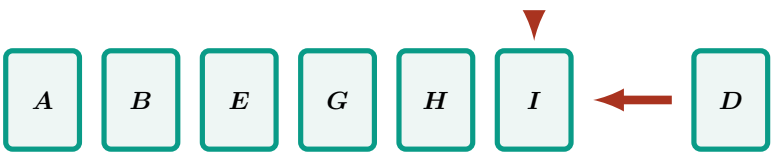

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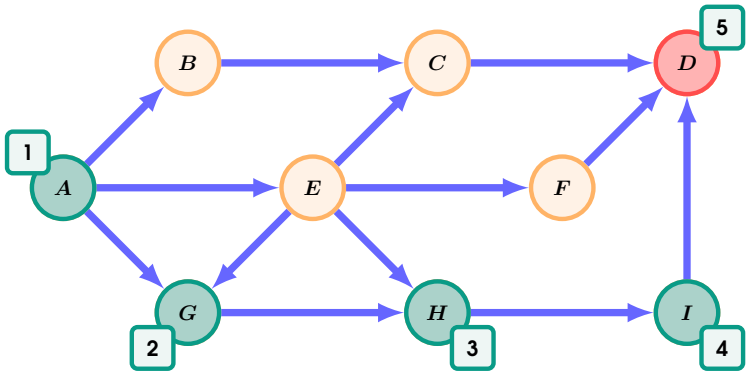

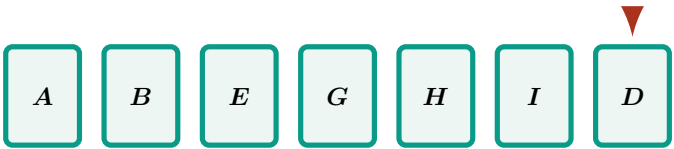

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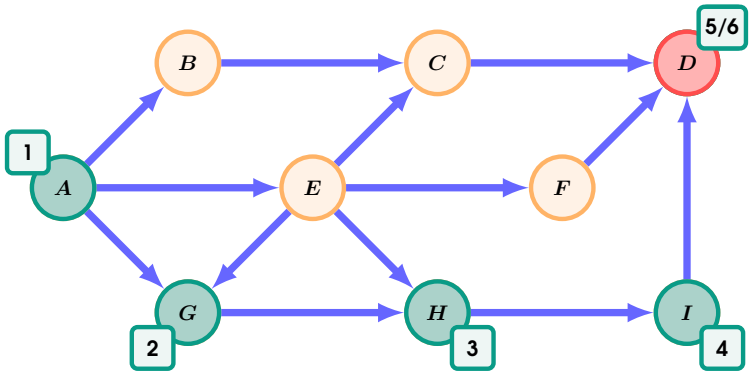

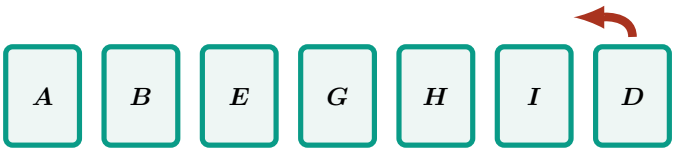

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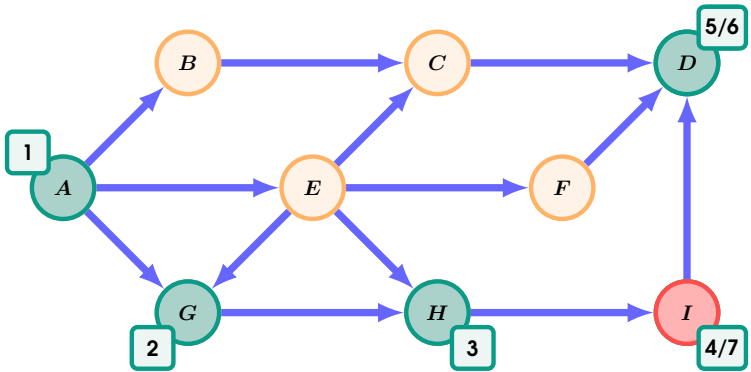

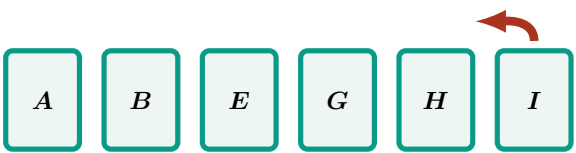

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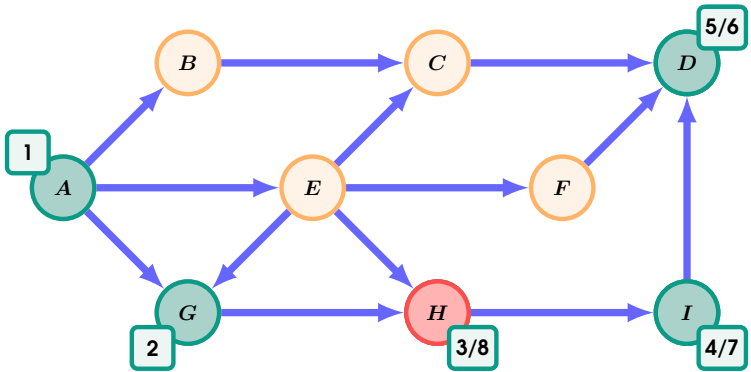

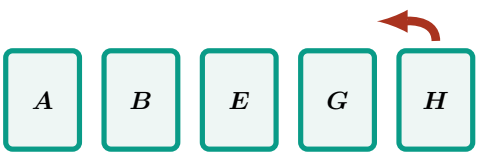

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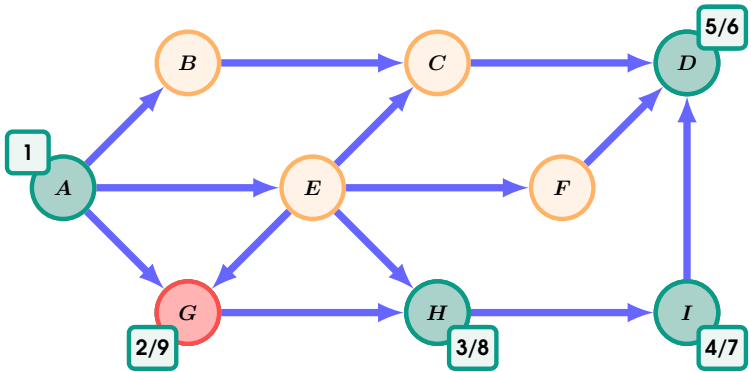

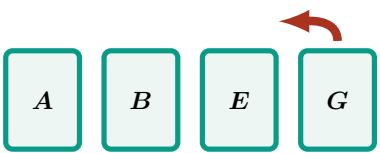

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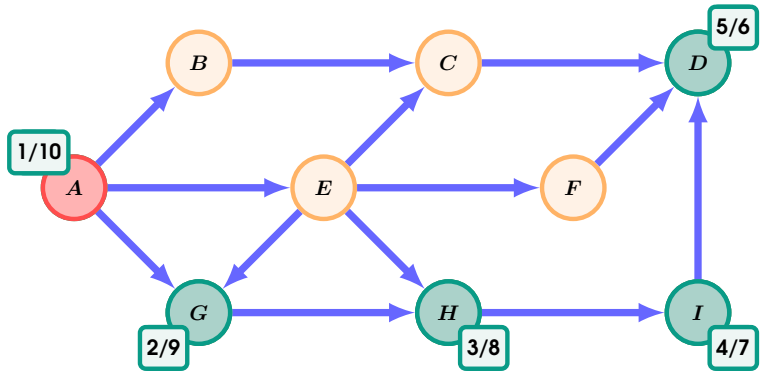

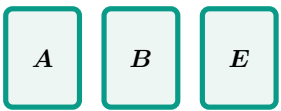

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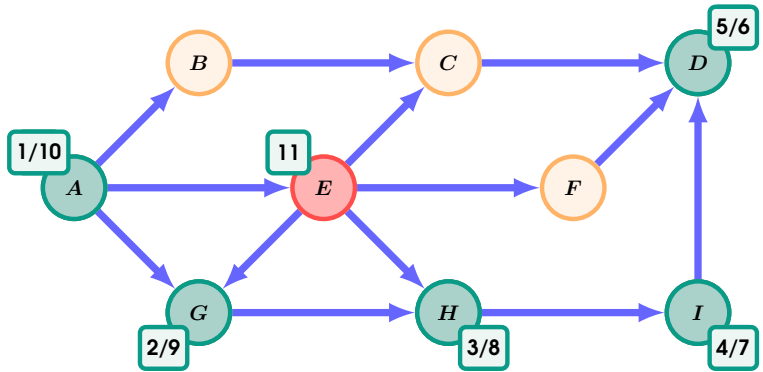

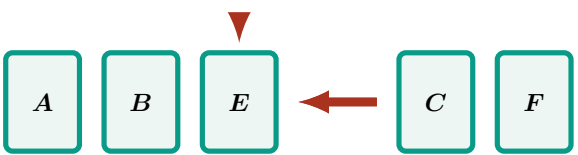

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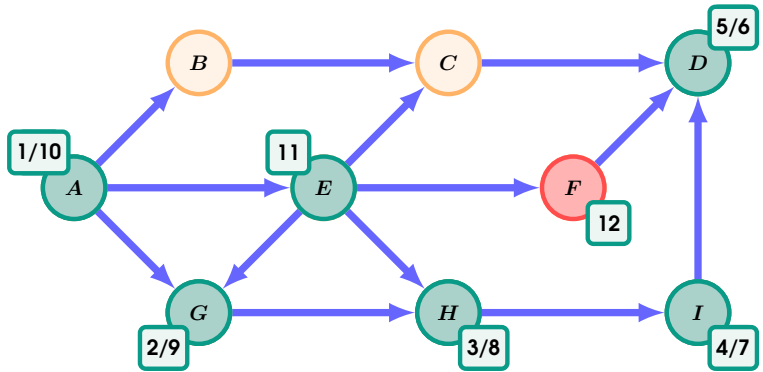

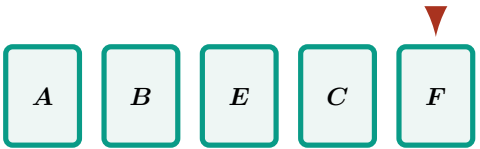

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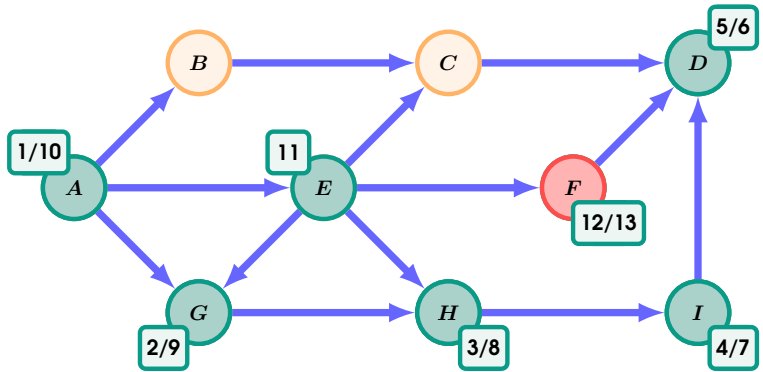

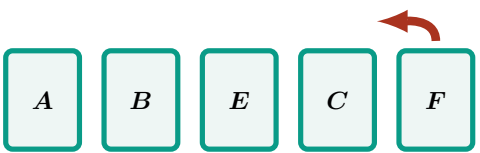

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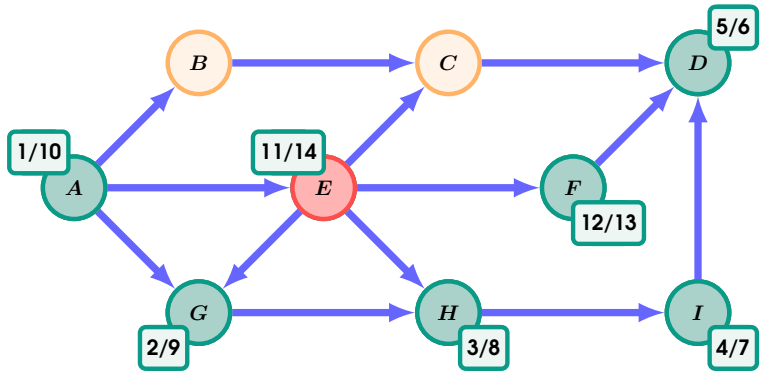

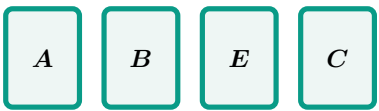

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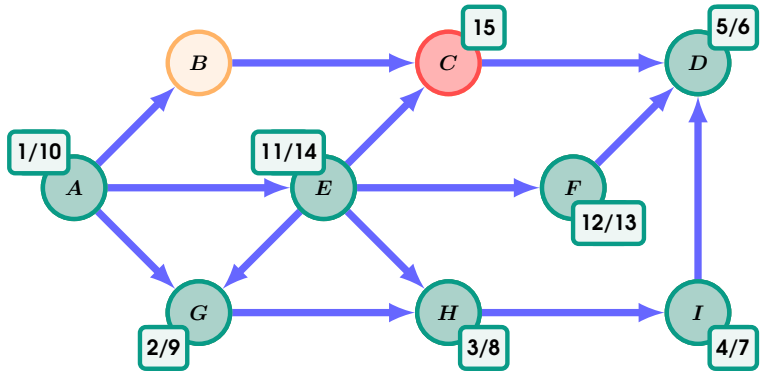

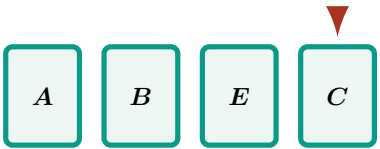

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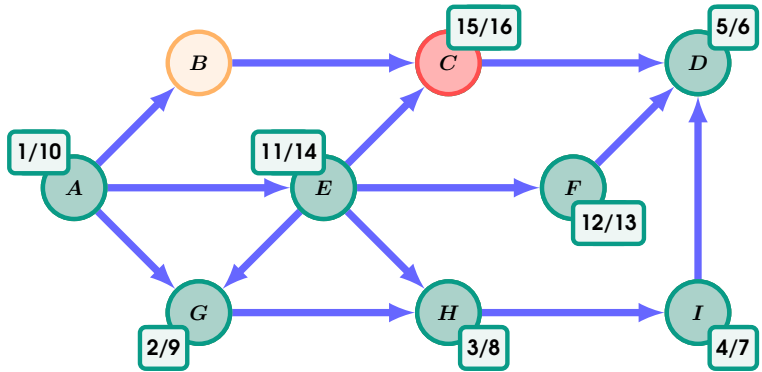

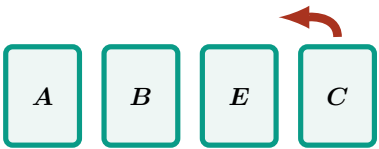

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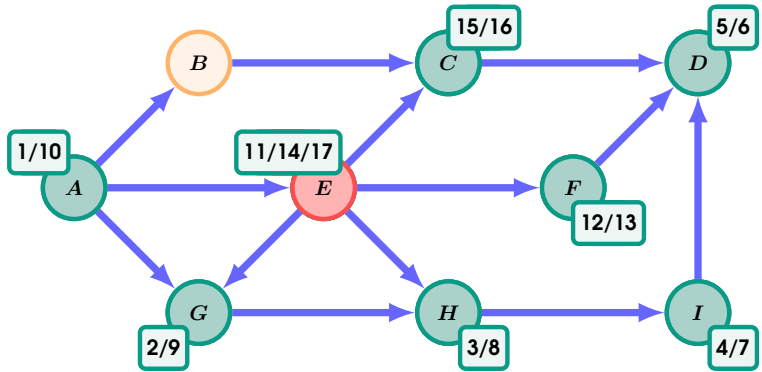
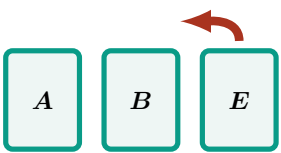
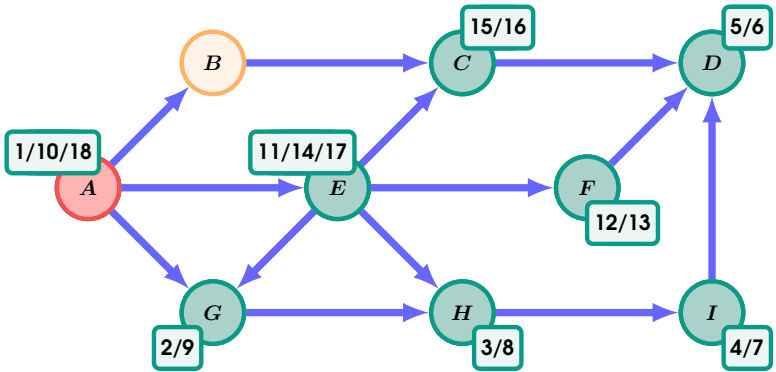
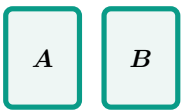


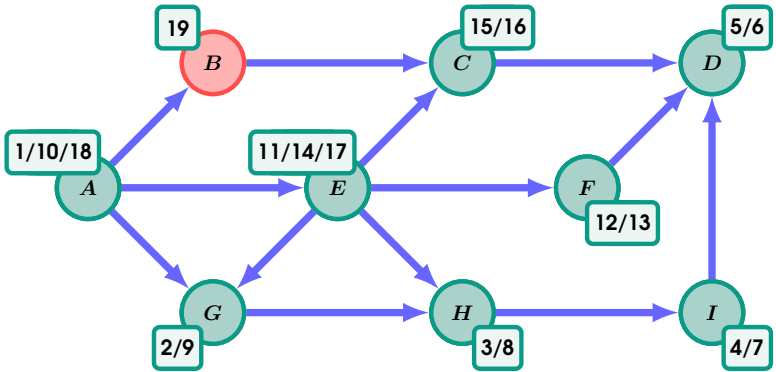


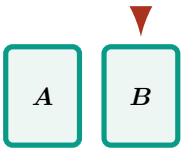
Depth-First-Search



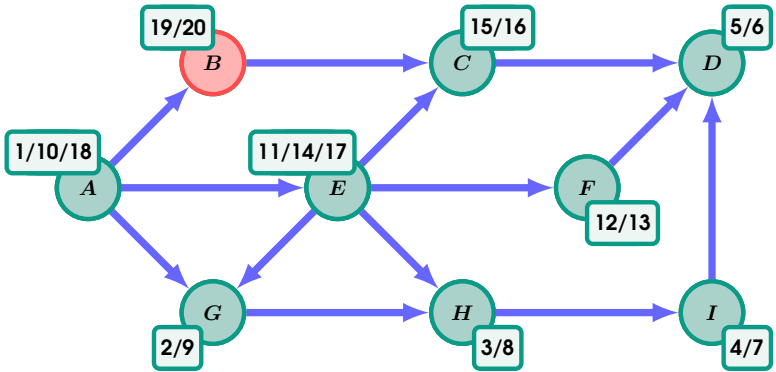


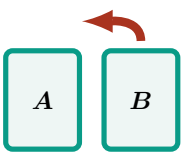
Depth-First-Search



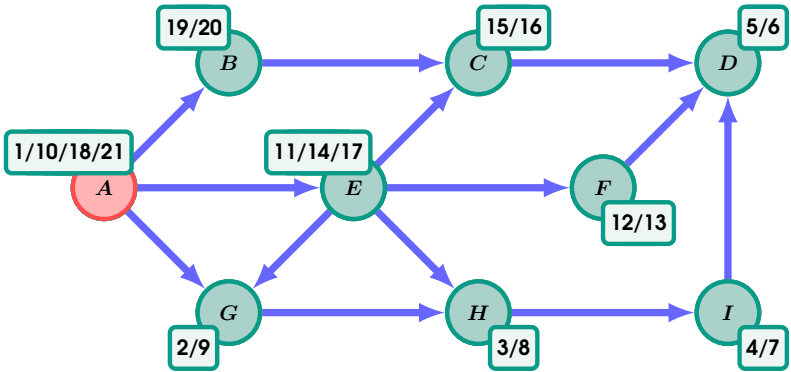


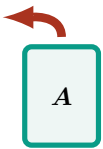
Depth-First-Search



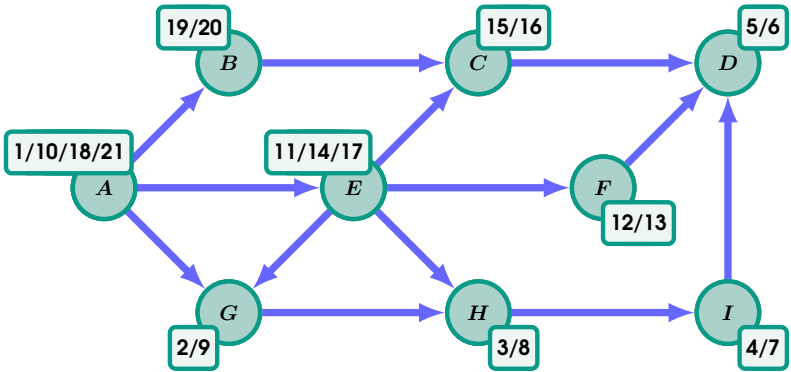


Depth-First-Search





Depth-First-Search



Stack Empty so Terminate!

Depth-First-Search

Theorem The termination time for Depth-First-Search is $O(|V| + |E|)$.

Proof:

- The proof is almost identical to that of the Breadth-First-Search, just replacing the **queue** for a **stack**.

Q.E.D.

End of Slides!