5CCS2FC2: Foundations of Computing II

Approximation Algorithms & the Travelling Salesman Problem

Week 7

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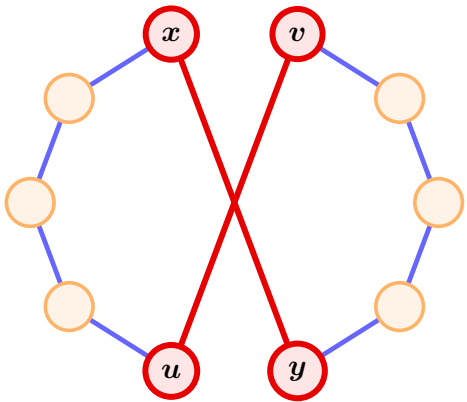
The 2OPT Approximation Algorithm

The 2OPT approximation algorithm

- **The 2OPT swap move:**

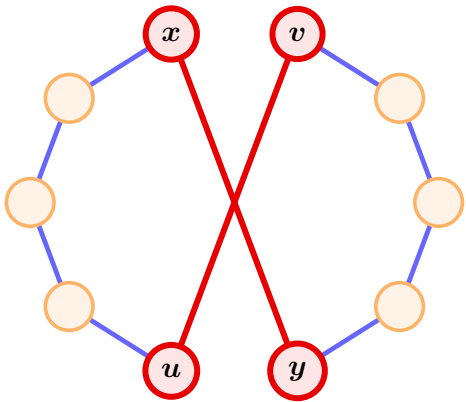
Step 1) Given a route $\boldsymbol{H}$, choose two *non-adjacent* edges in the cycle:

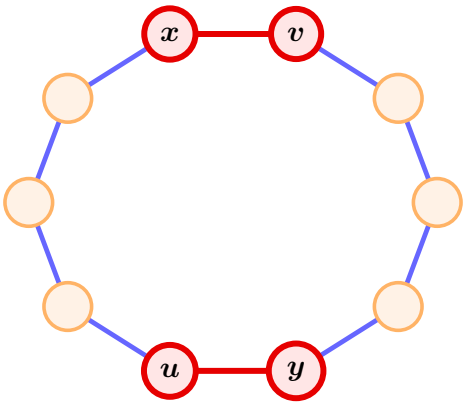


The 2OPT approximation algorithm

- **The 2OPT swap move:**

Step 2) Compare the *weight* of $d(x, y) + d(u, v)$ with $d(x, v) + d(u, y)$.

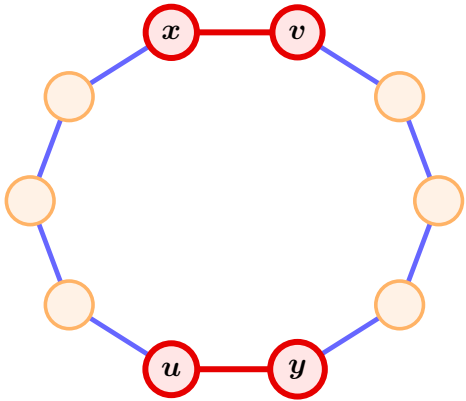




The 2OPT approximation algorithm

- **The 2OPT swap move:**

  Step 3) Replace the two edges whenever this **reduces** the total weight.

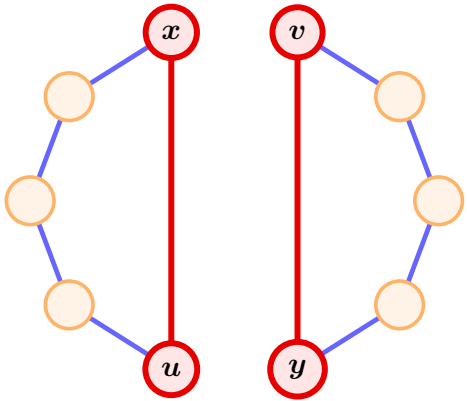


$$\underbrace{d(x,v) + d(u,y)}_{\text{after swap}} \quad < \quad \underbrace{d(x,y) + d(u,v)}_{\text{before swap}}$$

The 2OPT approximation algorithm

- **The 2OPT swap move:**

Step 4) Make sure that the swap does not **disconnect** the graph!



The 2OPT approximation algorithm

SWAP-MOVE (H) :

1. **For each** pair of *non-adjacent* edges (x, y) and (u, v) from H
    2. **If** $d(x, v) + d(u, y) < d(x, y) + d(u, v)$:
        3. Let H' be the result of replacing (x, y) and (u, v) with (x, v) and (u, y)
        4. **Return** H'
5. **Return** H unchanged if no improvement found

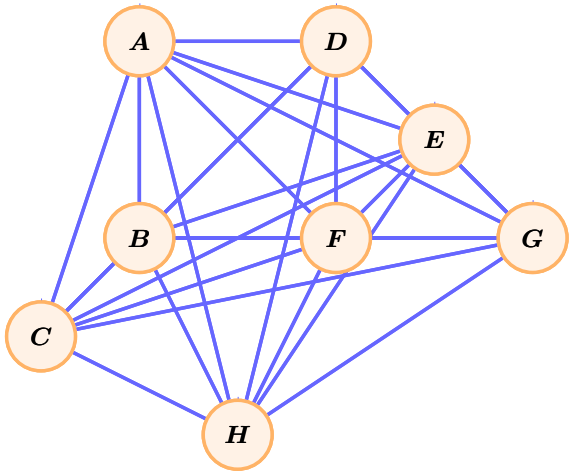
The 2OPT approximation algorithm

2OPT (G) :

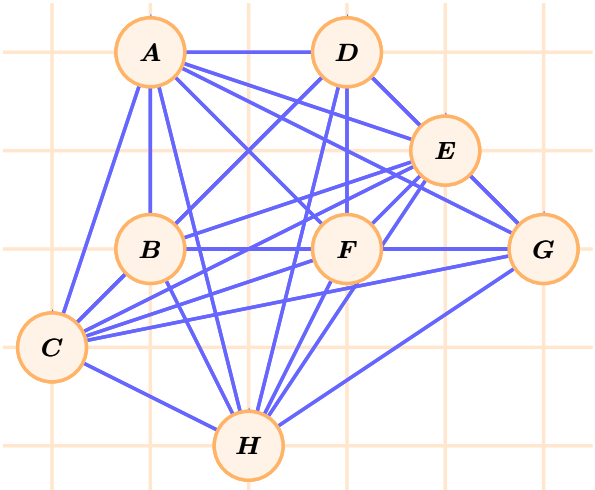
1. Contruct minimum spanning tree T_0 of G
2. Let H_0 be the *preorder traversal* of T_0
3. **While** H_i is not (local) optimal:
    4. $H_{i+1} \leftarrow$ SWAP-MOVE(H_i)
6. **Return** H

Example

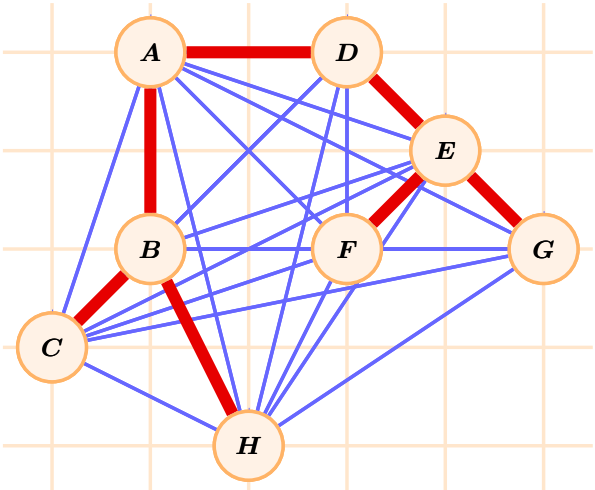
The 2OPT approximation algorithm



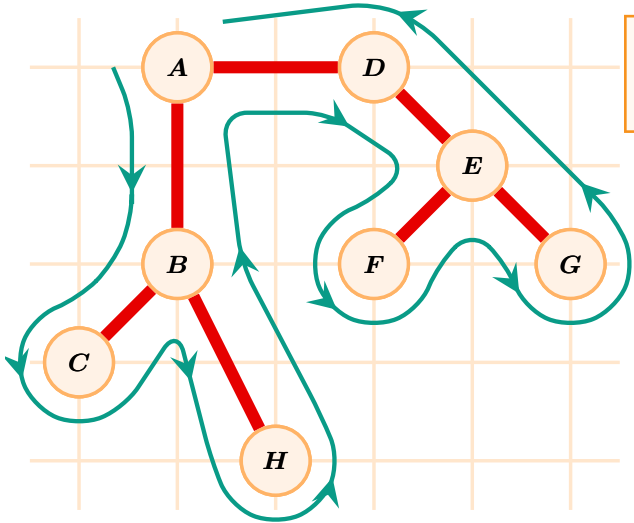
The 2OPT approximation algorithm



The 2OPT approximation algorithm



The 2OPT approximation algorithm

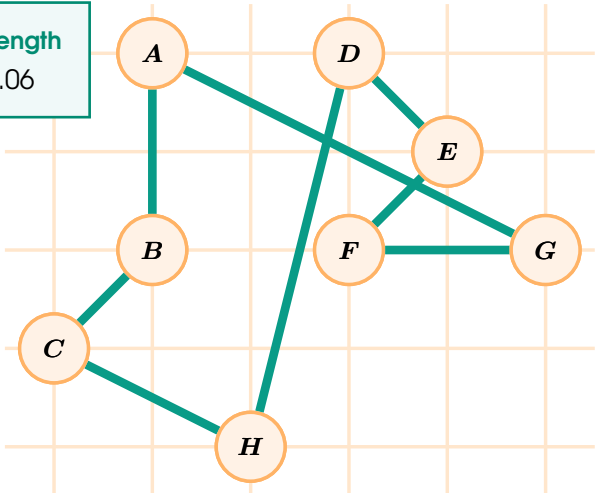


Preorder Traversal H_0

$A, B, C, H, D, E, F, G, A$

The 2OPT approximation algorithm

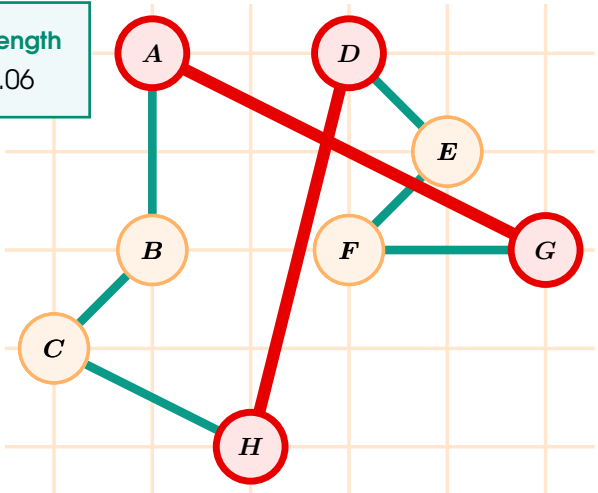
Cycle Length
≈ 19.06



Preorder Traversal H_0
$A, B, C, H, D, E, F, G, A$

The 2OPT approximation algorithm

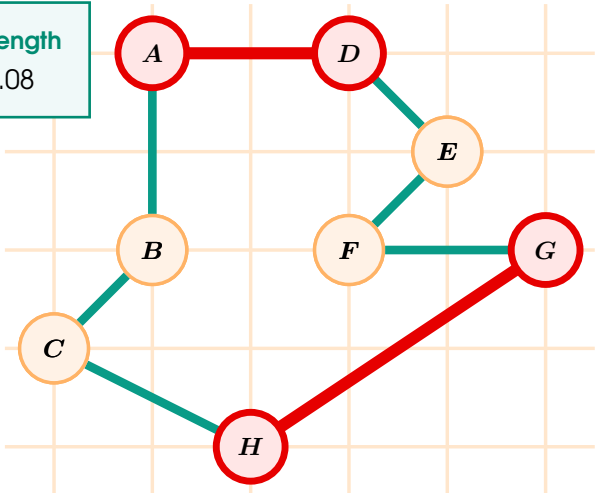
Cycle Length
≈ 19.06



Preorder Traversal H_0
$A, B, C, H, D, E, F, G, A$

The 2OPT approximation algorithm

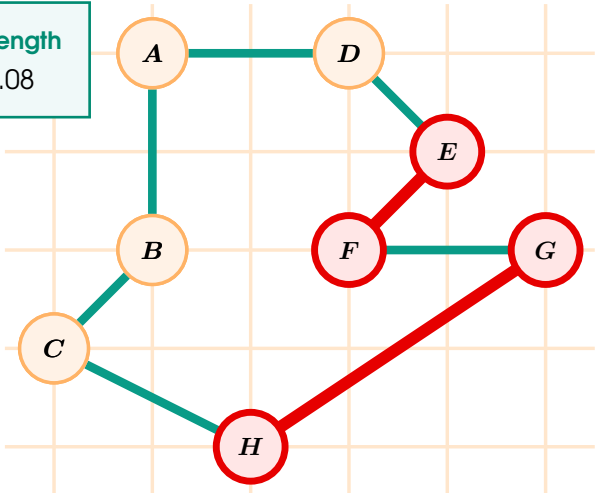
Cycle Length
≈ 16.08



Improved Traversal H_1
$A, B, C, H, G, F, E, D, A$

The 2OPT approximation algorithm

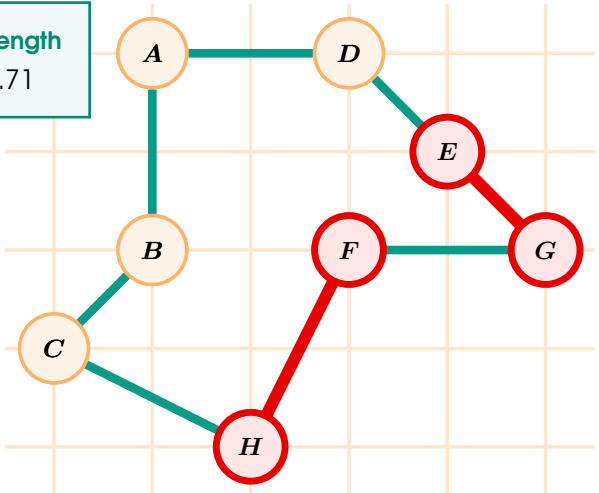
Cycle Length
≈ 16.08



Improved Traversal H_1
$A, B, C, H, G, F, E, D, A$

The 2OPT approximation algorithm

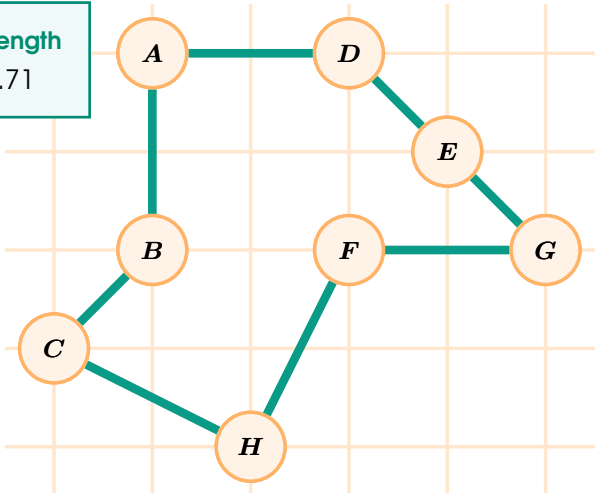
Cycle Length
≈ 14.71



Improved Traversal H_2
$A, B, C, H, F, G, E, D, A$

The 2OPT approximation algorithm

Cycle Length
≈ 14.71



(Local) Optimal Solution!
$A, B, C, H, F, G, E, D, A$

End of Slides!