Chapter 7

Economic Applications: Markets and Strategy

The concepts of Part I earn their keep in applications. This chapter analyzes the canonical models of imperfect competition and collective action—workhorses of industrial organization and public economics.

7.1 Cournot competition: quantities

Recall the setting (Examples 4.2, 6.3): n firms simultaneously choose outputs $q_i \geq 0$; inverse demand is $P(Q) = a - Q$ with $Q = \sum_i q_i$; each firm has constant marginal cost $c < a$.

Proposition 7.1: Cournot oligopoly with n firms

The unique Nash equilibrium is symmetric with

$$q_i^* = \frac{a-c}{n+1}, \qquad Q^* = \frac{n}{n+1}(a-c), \qquad P^* = \frac{a+nc}{n+1}, \qquad u_i^* = \left(\frac{a-c}{n+1}\right)^2.$$

As $n \to \infty$: $P^* \to c$ (perfect competition), and industry profit vanishes.

Proof. Firm i maximizes $q_i(a - q_i - Q_{-i} - c)$ where $Q_{-i} = \sum_{j \neq i} q_j$. The first-order condition $a - 2q_i - Q_{-i} - c = 0$ gives the best response $q_i = \frac{a-c-Q_{-i}}{2}$. Summing FOCs over i: $n(a - c) - (n+1)Q^* = 0$, so $Q^* = \frac{n}{n+1}(a-c)$; symmetry and substitution yield the rest. Uniqueness follows because best responses are linear with slope $-\frac{1}{2}$ in aggregate opponents' output, so the system has a unique solution. □

Key Idea: Competition as a continuum

Cournot interpolates smoothly between monopoly ($n = 1$: $Q^* = \frac{a-c}{2}$, the monopoly output) and perfect competition ($n \to \infty$: price equals marginal cost). Market structure maps into market power via a strategic model—the founding insight of industrial organization.

7.2 Bertrand competition: prices

Now let two identical firms set *prices*; consumers buy from the cheaper firm (splitting demand $D(p)$ at ties), and each firm has marginal cost c.

Proposition 7.2: Bertrand paradox

The unique Nash equilibrium has both firms pricing at marginal cost: $p_1 = p_2 = c$, with zero profits—even with only *two* firms.

Proof sketch. No equilibrium can have $\min(p_1, p_2) > c$: the higher-priced firm (or either firm, if tied) can undercut by a penny and capture the whole market at nearly the same margin. Prices below c yield losses, and asymmetric prices above cost invite undercutting. Checking $p_1 = p_2 = c$: any deviation upward sells nothing; downward loses money. □

The stark contrast with Cournot ($n = 2$ gives substantial profits) teaches an important modeling lesson: *the choice of strategic variable matters.* Real markets escape the Bertrand trap through capacity constraints (Kreps–Scheinkman: capacity choice then price competition reproduces Cournot outcomes), product differentiation, search costs, or repeated interaction (Chapter 9).

Example 7.1: Bertrand with differentiated products

Let demands be $q_1 = 1 - p_1 + bp_2$ and $q_2 = 1 - p_2 + bp_1$ with $0 < b < 1$ (imperfect substitutes), costs zero. Firm i maximizes $p_i q_i$; the FOC gives $\text{BR}_i(p_j) = \frac{1+bp_j}{2}$. Note best responses *increase* in the rival's price: prices are *strategic complements* (while Cournot quantities are *strategic substitutes*). Equilibrium: $p^* = \frac{1}{2-b}$, rising in b. Differentiation softens competition.

7.3 Public goods and free-riding

Example 7.2: Voluntary contribution to a public good

Each of n citizens has wealth w and chooses a contribution $g_i \in [0, w]$ to a public good. Payoffs: $u_i = w - g_i + m\sum_{j=1}^{n} g_j$ where $\frac{1}{n} < m < 1$ (the *marginal per capita return*). Since $\partial u_i/\partial g_i = m - 1 < 0$, contributing nothing is a *strictly dominant strategy*; the unique equilibrium is $g_i = 0$ for all. Yet full contribution would give everyone $w(nm) > w$: an n-player Prisoner's Dilemma. Laboratory subjects typically contribute 40–60% at first, decaying with repetition—and contributions are dramatically sustained if players can *punish* free-riders at a cost (Fehr–Gächter 2000), previewing the role of sanctions in Chapter 9.

7.4 Elections: the median voter

Proposition 7.3: Median voter theorem (Hotelling–Downs)

Two office-motivated candidates simultaneously choose platforms on a policy line; voters vote for the closer platform, and voter ideal points have median x_m. The unique Nash equilibrium has both candidates locating at x_m.

Proof sketch. If a candidate stands at $x \neq x_m$, the rival can locate between x and x_m (slightly on the median side) and capture more than half the electorate. Only (x_m, x_m) resists all such deviations, since deviating from the median concedes a majority to the opponent. □

The model explains the centrist pull in two-party systems—and its failures (primaries, turnout, multi-dimensional policy, entry of third candidates) map exactly onto its assumptions,

illustrating how a sharp game-theoretic benchmark organizes empirical debate.

7.5 Congestion: selfish routing

Example 7.3: Pigou's example

A unit mass of drivers travels from A to B via two roads: road 1 has fixed travel time 1; road 2 has travel time x equal to the fraction x of drivers using it. Equilibrium: drivers switch to road 2 until it is as slow as road 1, i.e. $x = 1$ (everyone on road 2), with average travel time 1. The social optimum splits traffic ($x = \frac{1}{2}$), for average time $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$. The ratio $\frac{1}{3/4} = \frac{4}{3}$ is the *price of anarchy*, and it is the worst possible for affine congestion costs (Chapter 17).

Example 7.4: Braess's paradox

Adding a zero-cost expressway to a congested network can make *every* driver worse off in equilibrium: the new link attracts traffic that congests links everyone must use. Removing roads has, on occasion, genuinely improved real urban traffic (documented cases in Seoul and New York). The paradox is impossible for a central planner—only selfish equilibrium behavior produces it—and is a vivid caution against "more options are always better" reasoning in strategic settings.

7.6 Strategic complements vs. substitutes: a unifying lens

Definition 7.1: Strategic complements and substitutes

In a two-player game with ordered action sets and smooth payoffs, actions are *strategic complements* if best responses are increasing in the opponent's action ($\partial^2 u_i / \partial a_i \partial a_j > 0$), and *strategic substitutes* if decreasing.

- **Substitutes** (Cournot quantities, public-good contributions, fishing effort): rivals' aggression makes you retreat. Comparative statics are moderate; free-riding is the typical pathology.
- **Complements** (Bertrand prices, bank runs, currency attacks, adoption of platforms, arms races): rivals' aggression makes you more aggressive. Multiplicity of equilibria, tipping, and self-fulfilling crises are the typical phenomena.

This classification (formalized in supermodular game theory—Topkis, Milgrom–Roberts, Vives) often predicts the *qualitative* behavior of a model before any computation.

Exercises

Exercise 7.1: Cournot with asymmetric costs

Two firms have costs $c_1 < c_2$, demand $P = a - Q$. Find the Nash equilibrium quantities and profits (assume both produce). Show the low-cost firm produces more, and find the cost gap at which firm 2 exits ($q_2^* = 0$).

Exercise 7.2: Bertrand with asymmetric costs

Two firms with costs $c_1 < c_2$ compete in prices for a market of unit demand with reservation price $v > c_2$. Characterize equilibrium outcomes (be careful: with continuous prices, use the convention that the low-cost firm wins ties, or work with discrete price grids and take limits). Who serves the market and at what price?

Exercise 7.3: Tullock rent-seeking contest

Two lobbyists spend $x_1, x_2 \geq 0$ competing for a prize V; lobbyist i wins with probability $x_i/(x_1 + x_2)$ (and $\frac{1}{2}$ if both spend 0). Payoffs are $\frac{x_i}{x_1+x_2}V - x_i$. Find the symmetric equilibrium and show total dissipation $x_1^* + x_2^* = V/2$: half the prize's value is burned in the contest.

Exercise 7.4: Bank run as coordination

Two depositors each placed D in a bank that invested in an illiquid asset. Each may Withdraw now or Wait. If both wait, the investment matures and each gets $R > D$. If one withdraws and the other waits, the withdrawer recovers D in full while the waiter receives less than D (liquidation loss). If both withdraw, each gets some amount strictly between those two (partial recovery). Set up a payoff matrix with concrete numbers satisfying these orderings, find both pure equilibria, and explain why deposit insurance eliminates the bad one (Diamond–Dybvig 1983, Nobel 2022).